# ANALYSIS OF DRILLING FLUID RHEOLOGY AND TOOL JOINT EFFECT TO REDUCE ERRORS IN HYDRAULICS CALCULATIONS

A Dissertation

by

MARILYN VILORIA OCHOA

Submitted to the Office of Graduate Studies of
Texas A&M University
in partial fulfillment of the requirements for the degree of

DOCTOR OF PHILOSOPHY

August 2006

Major Subject: Petroleum Engineering

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August 2006

Major Subject: Petroleum Engineering

# ABSTRACT

Analysis of Drilling Fluid Rheology and Tool Joint Effect to Reduce Errors in Hydraulics Calculations. (August 2006)

Marilyn Viloria Ochoa, B.S., Zulia University;

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Chair of Advisory Committee: Dr. Hans C. Juvkam-Wold

This study presents a simplified and accurate procedure for selecting the rheological model which best fits the rheological properties of a given non-Newtonian fluid and introduces five new approaches to correct for tool joint losses from expansion and contraction when hydraulics is calculated. The new approaches are enlargement and contraction (E&C), equivalent diameter (ED), two different (2IDs), enlargement and contraction plus equivalent diameter (E&C+ED), and enlargement and contraction plus two different IDs (E&C+2IDs).

In addition to the Newtonian model, seven major non-Newtonian rheological models (Bingham plastic, Power law, API, Herschel-Bulkley, Unified, Robertson and Stiff, and Casson) provide alternatives for selecting the model that most accurately represents the shear-stress/shear-rate relationship for a given non-Newtonian fluid.

The project assumes that the model which gives the lowest absolute average percent error ($E_{AAP}$) between the measured and calculated shear stresses is the best one for a given non-Newtonian fluid.

The results are of great importance in achieving correct results for pressure drop and hydraulics calculations and the results are that the API rheological model

(RP 13D) provides, in general, the best prediction of rheological behavior for the mud samples considered ($E_{AAP}$=1.51), followed by the Herschel-Bulkley, Robertson and Stiff, and Unified models. Results also show that corrections with E&C+2IDs and API hydraulics calculation give a good approximation to measured pump pressure with 9% of difference between measured and calculated data.

# DEDICATION

This dissertation is dedicated to my husband Jairo and my son Jairo Raul. They have always provided an atmosphere of love and understanding that helped me achieve my goal. I specifically dedicate my dissertation to my husband whose encouragement, love and good suggestions have made my dream of conquering a doctoral degree possible.

Also, I can not forget in this moment, my family in Venezuela.

They always have provided emotional support to help me finish this goal.

Finally, I want to dedicate this success in my life to my always remembered friend Sergei, who always was close to me when I needed him. Thank you my friend; I will follow all your advice, every single day of my life. You will be my good angel forever.

# ACKNOWLEDGMENTS

I would like to sincerely thank Dr. Juvkam-Wold for supervising this study. I am thankful to him for his ideas and for pointing out the proper direction for this work.

Thanks to Dr. Schubert for reading, correcting and giving me good ideas of how to present this report, and for his support and good mood since the beginning of this research.

I am also grateful to Dr. Ugaz and Dr. Valkó for reading this report and giving me good suggestions.

I am very thankful to Universidad del Zulia-CIED-PDVSA for their financial support provided for my studies.

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# CHAPTER I
# INTRODUCTION

The major applications of rheological properties for evaluating drilling fluid behavior are in solving problems of hole cleaning and hole erosion, suspension of cuttings, drilling fluid treatment, and hydraulics calculations. Hydraulics calculations are the focus of this project.

The viscosity of the drilling fluid must be known at all times because it determines the hydraulics in the well. The exact representation of this property differs depending on the type of fluid being pumped and rheological model being used for the evaluation of the fluid parameters. Many fluid properties depend on the system's rheology.

The rheology of dispersions, the most common drilling fluids today, is complex because they usually exhibit non-Newtonian behavior. Non-Newtonian fluids do not conform to a direct proportionality between shear stress and shear rate, and no single equation has been proved to describe exactly the rheogram of all such fluids. Shear stress in oil flied terms is analogous to the pump pressure.

In addition to the Newtonian model, this study examined seven major non-Newtonian rheological models (Bingham, Power law, API RP 13D[1], Herschel-Bulkley, Unified, Robertson and Stiff, and Casson) to identify additional alternatives for selecting the model that represents most accurately the shear-stress/shear-rate relationship for a given non-Newtonian fluid. This approach assumed that the model that gives the lowest absolute average percent error ($E_{AAP}$) between the measured and calculated shear stresses is the best one for a given non-Newtonian fluid.

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This dissertation follows the style and format of *SPE Journal*.

The first part of this study presents a simplified and accurate procedure for selecting the rheological model which best fits the rheological properties of a given non-Newtonian fluid.

The second part introduces five new approaches to correct for tool joint losses when hydraulics is calculated. These approaches are enlargement and contraction (E&C), equivalent diameter (ED), two different IDs (2IDs), enlargement and contraction plus equivalent diameter (E&C+ED), and enlargement and contraction plus two different IDs (E&C+2IDs).

Deep drilling needs high-strength drillpipe, which often has small-throated (internal upset) tool joints. These internal limitations cause flow losses that can be considerable. The pressure loss caused by entry into the tool joint is small compared with the exit losses.

On the other hand, the same problem can be experienced in the annulus between tool joint and casing due to the external upset of the tool joint. This space is narrower than the space between drillpipe and casing. The effect of expansion and contraction on the fluid flowing in the annulus is additional pressure loss.

The results of this research, methods to select the best rheological model and to estimate additional pressure loss from expansion and contraction of the fluid flowing through pipe and annuli, are of great importance in achieving correct results for pressure drop and hydraulics calculations.

Data from an offshore well showed that the API RP 13D model provides the best general prediction of rheological behavior for the mud samples considered ($E_{AAP}$=1.51). It was followed by Herschel-Bulkley, Robertson and Stiff, and the

Unified model. Also, correction with E&C+2IDs and the API hydraulics calculation gives a good approximation of measured pump pressure (9%).

### 1.1 Definition of the Problem

The drilling industry cannot without fail match calculated and actual pump pressures, $\Delta p_p$. For example, $\Delta p$ calculations using API RP13D with synthetic-based mud (SBM) can be off as much as 35%.[2] The possible reasons could be that friction pressure losses are functions of drilling fluid properties, which are functions of the rheological model, temperature, and well geometry.[3] As a result, current API RP13D equations seriously underestimate drillstring pressure losses, which account for the differences in pump pressure, $\Delta p$.

### 1.2 Importance

Many experimental studies deal with the flow of fluids though pipes and annuli for friction pressure loss calculations. Most of these studies have concentrated on rheological models, pipe roughness, and geometrical parameters. However, the effects of tool joints had yet been seriously investigated to estimate the friction pressure loss inside drillpipe and in the annulus. Additionally, selection of the best rheological model to obtain correct results for pressure drop and hydraulics have until now not been included in API RP13D.

This study of eight rheological models is expected to serve as a manual for the state of the art in rheology in drilling fluid, as well as in hydraulics calculation. This dissertation could also be used in an educational environment and for training purposes; it would help inform and educate the industry about rheology in drilling fluid and hydraulics calculation considering different rheological models as well as tool joint corrections.

# CHAPTER II
# BACKGROUND RESEARCH

## 2.1 Literature Review

Advances in the areas of drilling fluid rheology, tool joint effects, and hydraulics in drilling wells offer insight into the obstacles to choose appropriate equations.

### 2.1.1 Drilling Fluid Rheology

Most drilling fluid muds are non-Newtonian fluids, with viscosity decreasing as shear rate increases.[3]

Herzhaft *et al.*[4] showed that plastic viscosity is the parameter most affected by temperature changes. On deepwater wells, the cooling effect of the riser will result in higher plastic viscosity in the drilling fluid. Additionally, the length of the riser enhances the cooling effect during circulation and during trips, creating major changes in rheology if oil-based or synthetic mud is used. Changes in mud viscosity may also lead to problems with surge and swab, transmission of measurement-while-drilling (MWD) pulses, increased equivalent circulating density and variations in hole-cleaning efficiency.

Zamora and Power[2] detailed in their paper a new unified rheological model. The rheological parameters for this model are the plastic viscosity $(\mu_p)$, yield point $(\tau_y)$, and yield stress $(\tau_0)$. A fourth parameter, the ratio $\tau_0/\tau_y$, is a useful tool to help characterize fluids rheologically, although it is not necessary for solving the model. However, many RP 13D elements are still valid and in use, but some need to be updated. Mud rheology needs adjustment for downhole conditions, especially in ultradeepwater wells drilled with oil or synthetic mud.

Power and Zamora[5] showed that the ratio $\tau_0/\tau_y$ is a useful parameter to characterize fluids rheologically. The acceptable range of $\tau_0/\tau_y$ values is 0 to 1 for rheological models used in drilling. It will be better explained in Chapter III.

### 2.1.2 Pressure and Temperature Effect in the Rheology of Drilling Fluid

Politte[6] concluded from his analysis of rheological data for emulsion that drilling fluid yield point is not a strong function of pressure, and becomes progressively less so as temperature increases. The effects of temperature on the yield point, however, are difficult to predict as they require chemical particle effects.

Davison *et al.*[7] concluded from their study of rheological data obtained from a viscosimeter that the effect of low temperature on both oil-based mud (OBM) and synthetic mud (SBM) viscosity is quite pronounced. On the other hand, when pressure was increased at various temperatures, viscosity of both oil-based and SBMs increased, especially at higher shear rates. The pressure effects don't appear to be dependent on the temperature. **Fig. 2.1** shows some results.

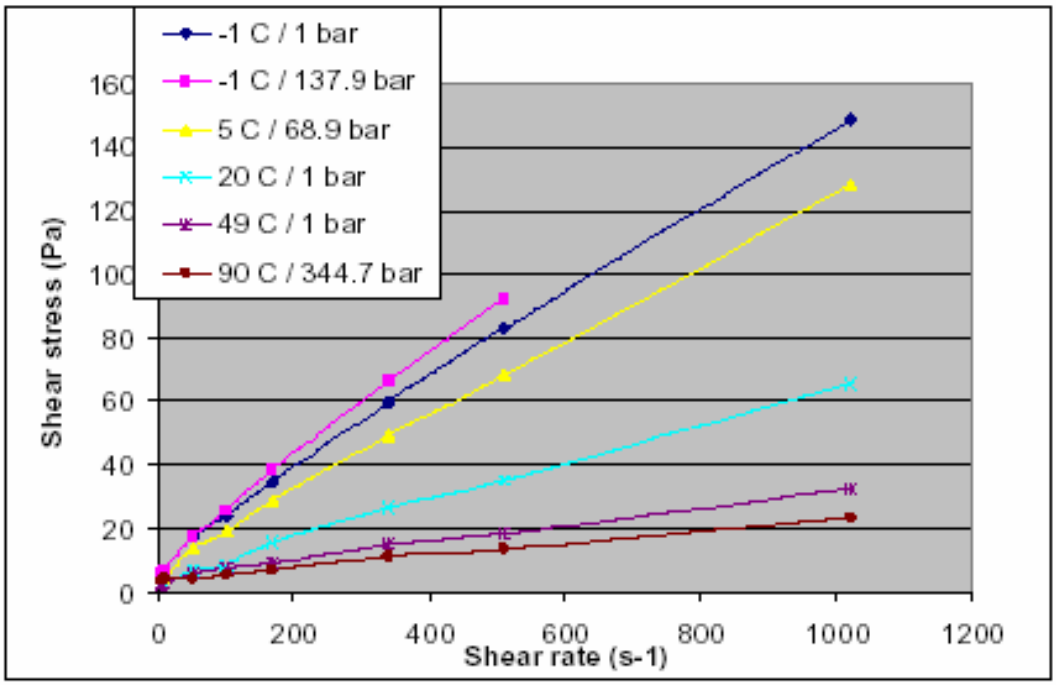



Fig. 2.1—Rheograms at various temperatures and pressures for unweighted oil- based mud, 80:20 oil/water ratio.[7]

Prediction of hydrostatic pressure requires pressure/volume/temperature (PVT) data for the mud in addition to an accurate simulation of the downhole temperature profile. The compressibility of a drilling fluid depends on its base fluid; the solids are incompressible.

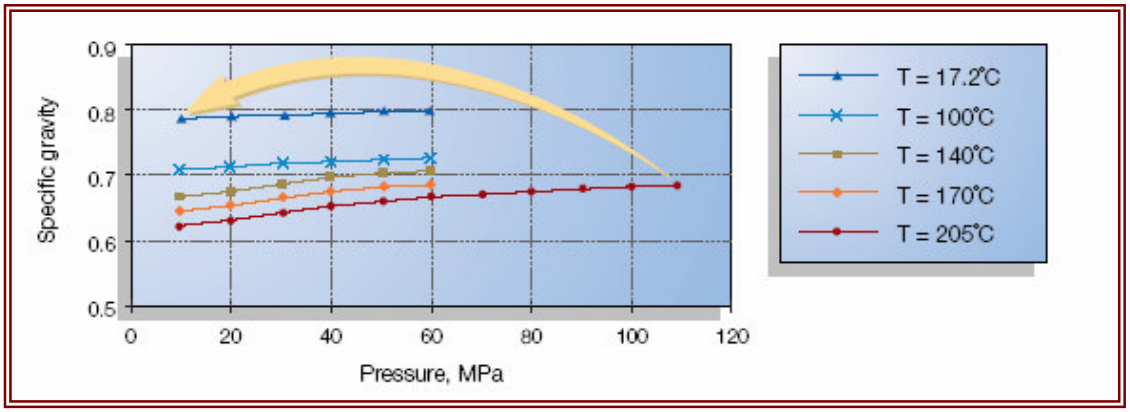



Fig. 2.2— Low-toxicity, biodegradable, organic-base fluid.[8]

**Fig. 2.2** is the PVT diagram for the low-toxicity, biodegradable, organic-base fluid of the ULTIDRILL system, which has been used at 395°F and at weights up to 19 ppg. The specific gravity of the base fluid under these conditions at a depth of 16,000 feet is 0.68. The same fluid (arrow) returned to surface temperature and pressure has a specific gravity of 0.79, a 14% decrease in base fluid density at total depth, which is important in computing static pressure.[8]

### 2.1.3 Hydraulics in Deepwater

Zamora and Power[2] evaluated the inability of API equations from RP 13D to match field data in critical drilling, because these equations have to incorporate the effects of temperature and pressure on SBM density and rheological properties.

### 2.1.4 Tool Joints

White and Zamora[9] established from a comparison between field and calculated data that one possible opportunity for discrepancies is increase in pressure caused by sudden contraction and expansion of the mud when passing through the tool joints, which is not considered in any published hydraulics calculation.

Denison[10] concluded that internally constricted drillstring elements can drastically affect the rig hydraulics. Also, the pressure loss caused by entry into the tool joint is small compared with the exit losses.

Yeon-Tae and Subhash[11] found that the effect of the presence of tool joints on the annular friction pressure is significant, and they proposed an accurate prediction method for annular pressure loss.

# CHAPTER III
# RHEOLOGY

## 3.1 Understanding Drilling Fluid Rheology

The term “rheology” means the study of the deformation and flow of matter, including such widely differing materials as asphalt, lubricants, paints, plastics and rubber, which gives some idea of the scope of the subject and also the numerous scientific disciplines which are likely to be involved.[12]

Currently, the scope is even wider. Significant advances have been made in bio-rheology, in polymer rheology, in suspension rheology, and in the chemical processing and oil industries.[12]

## 3.2 Components of Rheological Research

### 3.2.1 Rheometry

Rheometry is the science of reproducing deformation and measuring the consequences on materials of interest. A rheometer reproduces deformation under controlled conditions representative of those found in real production processes such as temperature and deformation rate.

### 3.2.2 Constitutive Equations

In practice, rheology has usually been restricted to the study of the fundamental relations, called constitutive relations, between force and deformation in materials, primarily liquid.[13]

## 3.3 Viscosity

Viscosity is traditionally regarded as a most important material property, and any practical study requiring knowledge of material response would automatically turn to the viscosity.[12]

The concept of viscosity was introduced by Newton's postulate, in which the shear-stress ($\tau$) was related to the velocity gradient, or shear rate ($\gamma$), through the equation:

$$\tau = \mu\gamma . \quad \text{.....3.1}$$

For Newtonian liquids, $\mu$ is sometimes called the coefficient of viscosity, but it is now more commonly referred to simply as the viscosity. Such a terminology is helpful within the context of rheology, since, for most liquids, $\mu$ is not a coefficient, but a function of the shear rate ($\gamma$).

### 3.3.1 Practical Ranges of Variables Which Affect Viscosity

The viscosity of real materials can be significantly affected by such variables as temperature and pressure, and it is clearly important for drilling fluid engineers to understand the way viscosity depends on such variables.[12]

For all liquids, viscosity decreases with increasing temperature and decreasing pressure. The strong temperature dependence of viscosity is such that, to produce accurate results, great care has to be taken with temperature control in viscometry. For liquids of higher viscosity, given their stronger viscosity dependence on temperature, even greater care has to be taken.[12]

The viscosity of liquids increases exponentially with isotropic pressure. Water below 30℃ is the only exception; the viscosity of water first decreases before eventually increasing exponentially. The changes are quite small for pressures differing from atmospheric pressure (14.7 psi). Therefore, for most practical purposes, the pressure effect is ignored by viscometer users. In some situations, however, this would not be justified. For example, the oil industry requires measurements of the viscosity of lubricants and drilling fluids at elevated pressures.[12]

### 3.4 The Shear-Dependent Viscosity of Non-Newtonian Liquids

In the vast majority of drilling fluids, viscosity decreases with increase in shear rate, giving rise to what is now generally called “shear-thinning” behavior although the terms “temporary viscosity loss” and “pseudoplasticity” have also been employed.

In some cases (although few in number) the viscosity increases with shear rate. Such behavior is generally called “shear-thickening,” although the term “dilatancy” has also been used.[12]

The very act of deforming a material can cause rearrangement of its microstructure such that the resistance to flow increases with shear rate.[11] Many shear-thinning fluids will exhibit Newtonian behavior at extreme shear rates, both low and high. These two extremes are sometimes known as the lower and upper Newtonian regions respectively. For such fluids, when the apparent viscosity is plotted against log of shear rate, we see a curve as shown in **Fig. 3.1.**

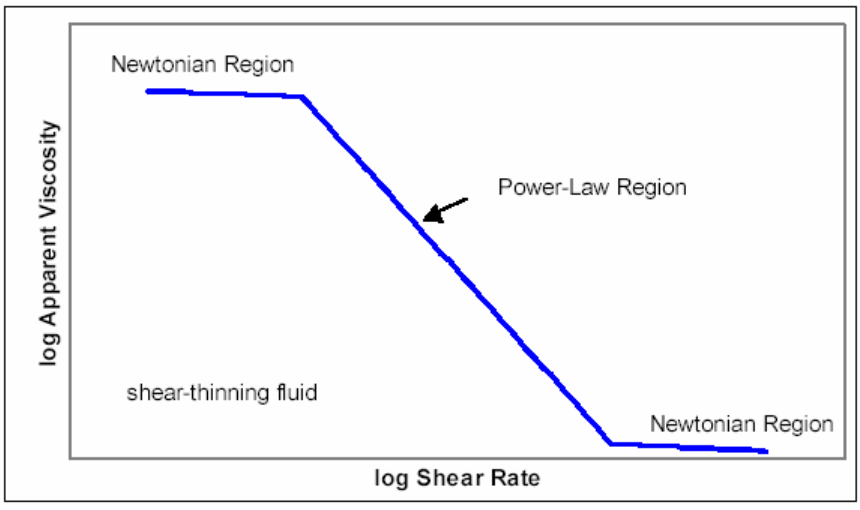



Fig. 3.1— Shear thinning or pseudoplastic fluid behavior (non-linear).[11]

The terms "first Newtonian region" and "second Newtonian region" have also been used to describe the two regions where the viscosity reaches constant values.[12]

### 3.5 Linear Viscoelasticity

During the latter half of the nineteenth century, scientists began to note that a number of materials showed time dependence in their elastic response. Today we call this time-dependent response “viscoelasticity.”[13]

The word “viscoelastic” means the simultaneous existence of viscous and elastic properties in a material. All real materials are viscoelastic; i.e., in all materials, both viscous and elastic properties coexist. The particular response of a sample in a given experiment depends on the time scale of the experiment in relation to a natural time of the material. Thus, if the experiment is relatively slow, the sample will appear to be viscous rather than elastic, whereas if the experiment is relatively fast, it will appear to be elastic rather than viscous. At intermediate time scales a mixed (viscoelastic) response is observed. An example of a common viscoelastic liquid is egg-white.[12]

### 3.6 Viscoplastic or “Yield Stress” Fluid

Another important type of non-Newtonian fluid is a viscoplastic or “yield stress” fluid. This is a fluid which will not flow when only a small shear stress is applied. The shear stress must exceed a critical value known as the yield stress, $\tau_0$, for the fluid to flow. For example, a tube of toothpaste should not flow at the slightest amount of shear stress; we need to apply an adequate force before the toothpaste starts flowing. So, viscoplastic fluids behave like solids when the applied shear stress is less than the yield stress. Once it exceeds the yield stress, the viscoplastic fluid will flow just like a fluid. Bingham plastics are a

special class of viscoplastic fluids that exhibit a linear behavior of shear stress against shear rate.

### 3.7 Time Effects in Non-Newtonian Liquids

We have so far assumed by implication that a given shear rate results in a corresponding shear stress, whose value does not change so long as the value of the shear rate is maintained. This is often not the case. The measured shear stress, and hence the viscosity, can either increase or decrease with time of shearing. Such changes can be reversible or irreversible.[12]

According to the accepted definition, a gradual decrease of the viscosity under shear stress followed by a gradual recovery of structure when the stress is removed is called “thixotropy.” The opposite type of behavior, involving a gradual increase in viscosity under stress, followed by recovery, is called “negative thixotropy” or “antithixotropy”.[11]

Thixotropy usually occurs in circumstances where the liquid is shear-thinning (in the sense that viscosity levels decrease with increasing shear rate, other things being equal). In the same way, antithixotropy is usually associated with shear-thickening behavior. **Fig. 3.2** shows the behavior to be expected from relatively inelastic colloidal materials with the shear rate increasing continuously and linearly in time from zero to some maximum value and then decreasing to zero in the same way.[12]

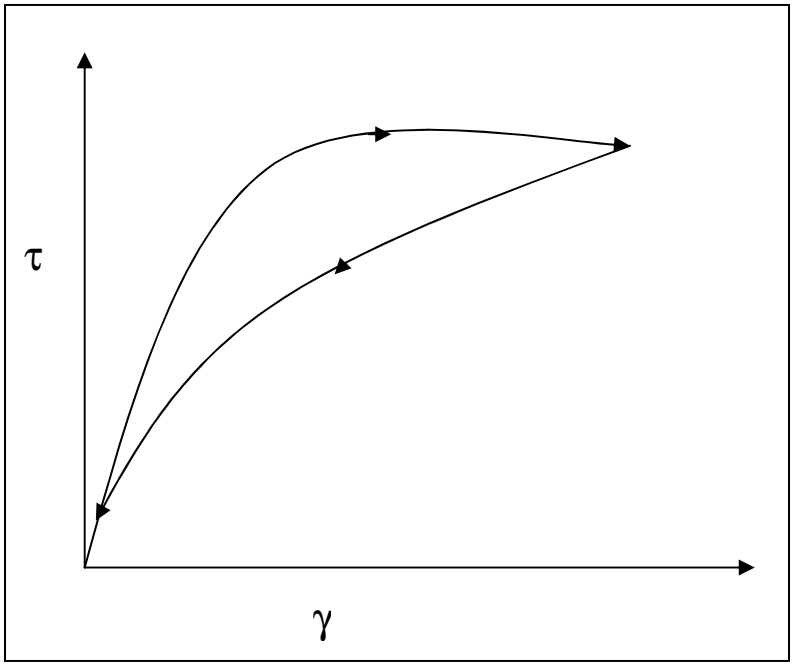



Fig. 3.2— Thixotropic effect (from Thivolle[3]).

The occurrence of thixotropy implies that the flow history must be taken into account when making predictions of flow behavior. For instance, flow of a thixotropic material down a long pipe is complicated by the fact that the viscosity may change with distance down the pipe.[13]

The bentonite suspensions used in drilling fluids are often thixotropic because the breakage and restoring of the network are reversible and not instantaneous, so that fluid properties are governed by different levels of structure.

### 3.8 Rheology of Suspensions

A suspension, or more broadly dispersion, consists of discrete particles randomly distributed in a fluid medium. Generally we divide suspensions into three categories: solid particles in a liquid medium (often the word “suspension”

is restricted to this meaning), liquid droplets in a liquid medium (or an emulsion), and gas in a liquid.[13]

Adding a particle does not simply change the magnitude of viscosity; it also can introduce all the known deviations from Newtonian behavior.

The first Newtonian plateau at low shear rate is followed by the Power-law shear- thinning region and then by a flattening out to the upper (second) Newtonian plateau. At some point, usually in this upper Newtonian region, viscosity can increase for suspensions of solid particles, given the appropriate conditions. In certain situations the first Newtonian plateau is sometimes so high as to be inaccessible to measurement. In such cases the low shear rate behavior is often described by an apparent yield stress.[12]

### 3.8.1 Forces Acting on Particles Suspended in a Liquid

Three kinds of forces coexist to various degrees in flowing suspensions. First, are those of colloidal origin that arise from interactions between the particles. These are controlled by properties of the fluid such as polarizability, but not by viscosity. These forces can result in an overall repulsion (electrostatic charges) or attraction between the particles. The Brownian force is strongly size dependent, ensures that the particles are in constant movement, and any description of the spatial distribution of the particles is a time average. The viscous forces acting on the particles are proportional to the local velocity difference between the particle and the surrounding fluid. For this reason, suspension viscosity is usually considered as the viscosity relative to that of the continuous phase. Clearly, the rheology measured macroscopically is strongly depending on this microstructure consideration.[12]

### 3.9 Oil-Based Mud Rheological Properties as a Function of Temperature and Pressure

Drilling fluids are called “oil-based mud” (OBM) if the continuous phase is composed of a liquid hydrocarbon. Diesel usually is used for the oil phase because of its viscosity characteristic, low flammability, and low solvency for rubber. In addition to diesel oil, weathered crude oils and various refined oils have been used as the oil phase for OBMs.[14]

Recently, several mineral oils have been developed that have a lower toxicity than diesel oil. These oils were developed to help solve the potential pollution problems associated with use of oil muds in a marine environment. The chosen oil should exhibit an acceptable viscosity over the entire range of temperatures and pressures to be encountered in the well. The effects of temperature and pressure on the viscosity of diesel oil are shown in **Fig. 3.3**.

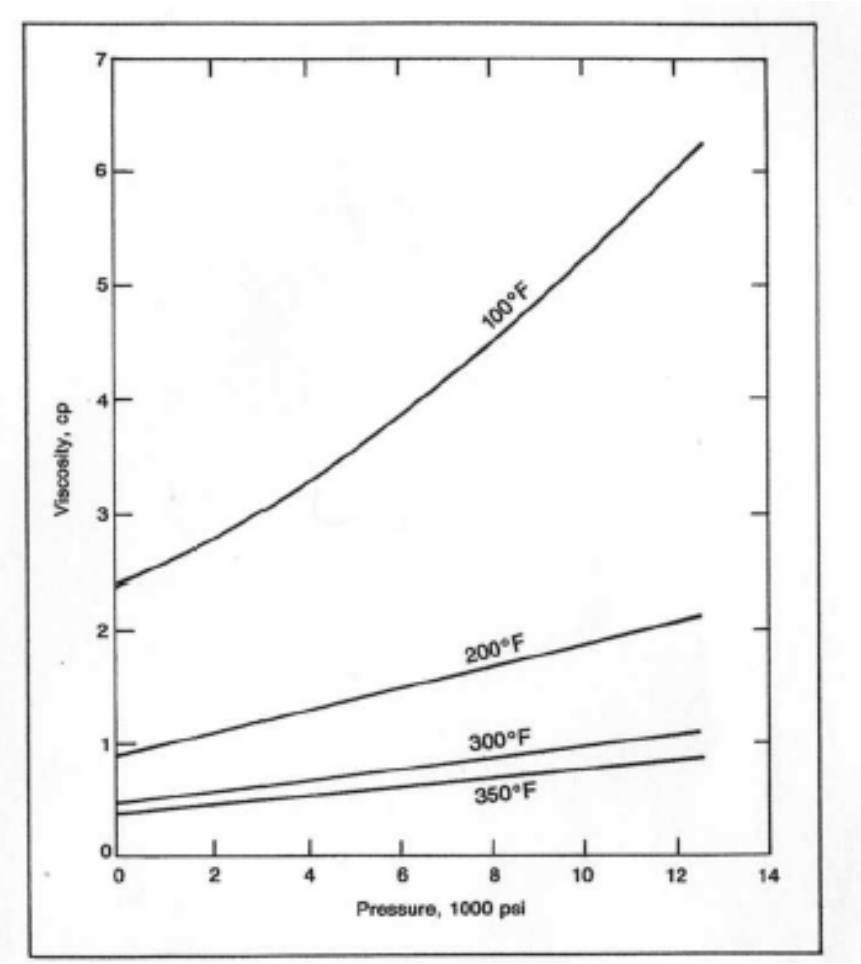



Fig. 3.3—Effect of temperature and pressure on the viscosity of diesel oil (from Lummus[15]).

**Figs. 3.4 and 3.5** show real data for OBM from a Fann 70 viscometer. This data was obtained from Bogotá Technical Center-Colombia. The sample used diesel as the liquid phase in an OBM of 80:20 oil/water ratio.

The figures show how plastic viscosity and yield point (rheological properties from the Bingham plastic model) behave with variation of pressure and temperature.

Increasing pressure at constant temperature increases plastic viscosity and yield point. When temperature increases at constant pressure, the properties decrease.

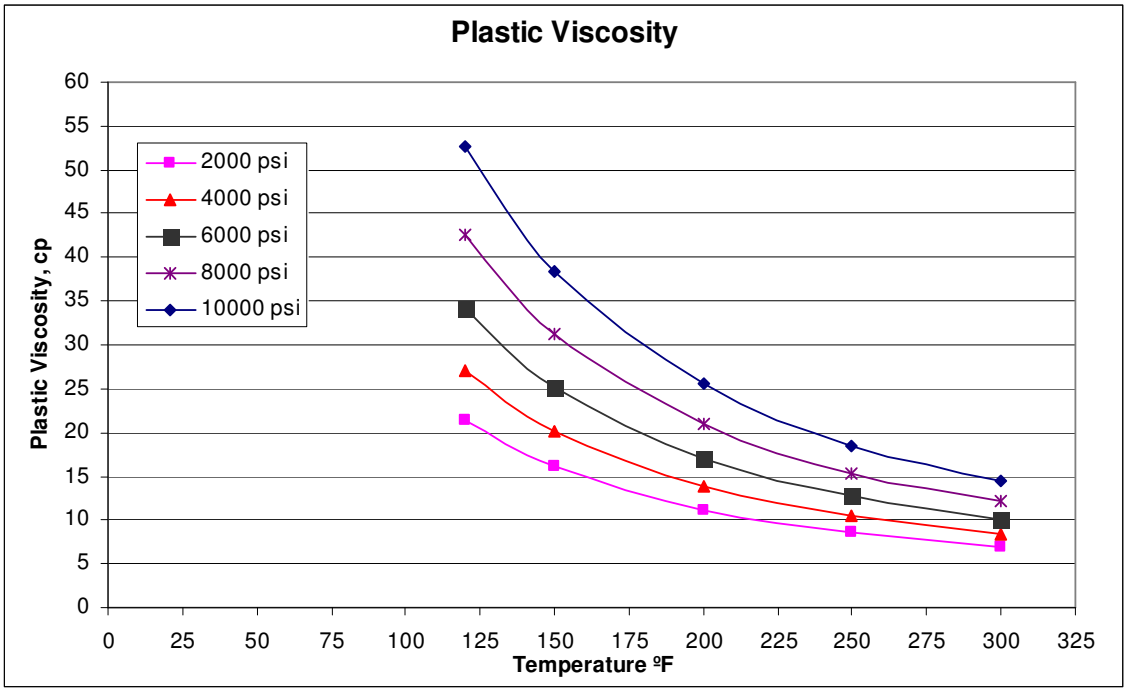



Fig. 3.4—Effect of pressure and temperature on plastic viscosity (from Bogotá Technical Center-Colombia[†]).

[†] Data provided by Ecopetrol, Bogotá- Colombia. 2005.

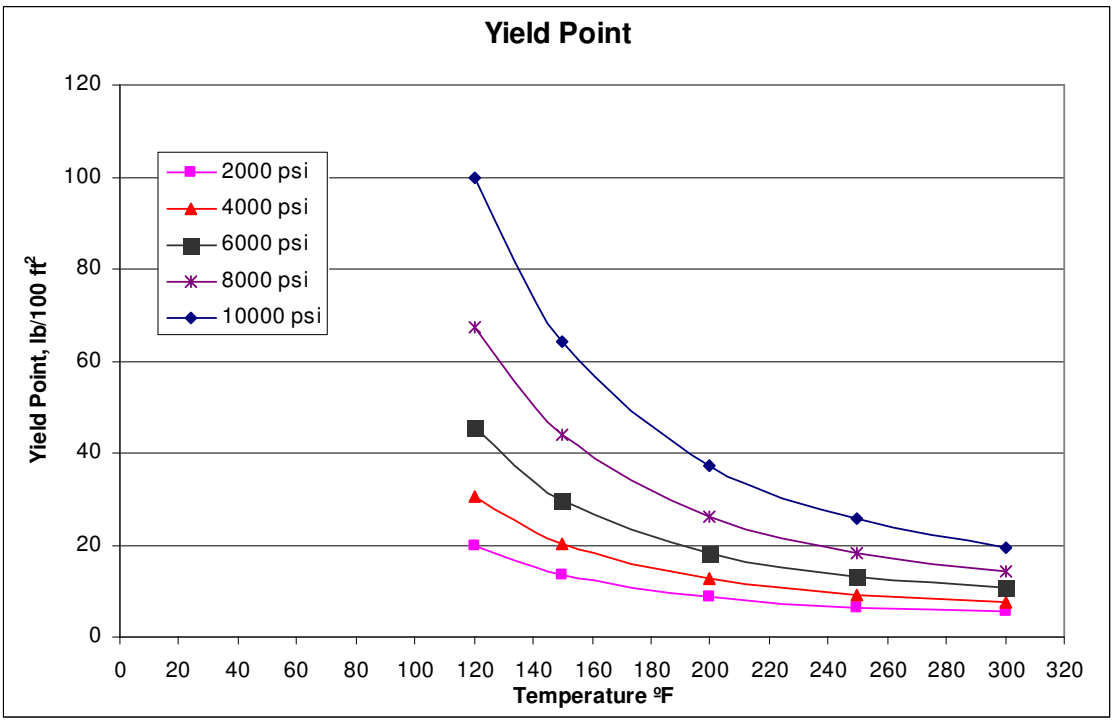



Fig. 3.5—Effect of pressure and temperature on yield point.
(from Bogotá Technical Center-Colombia‡).

In the deepwater environment, water temperatures easily reach 40°F (5°C) and below. This low-temperature environment effectively cools down the drilling fluid, significantly increasing fluid viscosity, which in turn impacts equivalent circulating densities. Narrow drilling margins (i.e., the window between fracture gradient and pore pressure) encountered in deepwater drilling operations often make such rheological increases intolerable, resulting in severe losses of SBM and thus significant increasing fluid cost and rig time. **Fig. 3.6** shows that for low circulation rates the temperature drops very rapidly and the fluid enters the wellbore almost at the same temperature to sea water profile.[16]

‡ Data provided by Ecopetrol, Bogotá- Colombia. 2005.

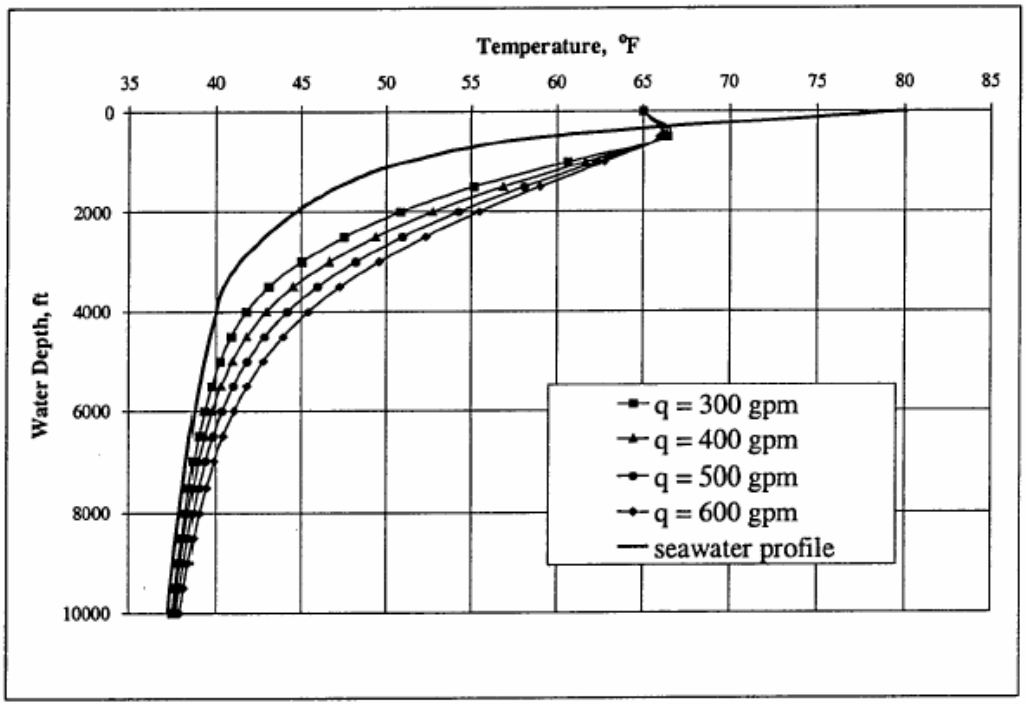



Fig. 3.6—Effect of mud flow-rate on the drillstring fluid temperature above seafloor (from Lima[16]).

**Fig. 3.7** shows low-temperature PVT data taken on a Huxley-Bertram unit for an IO1618 fluid commonly used to formulate deepwater, SBMs.[17]

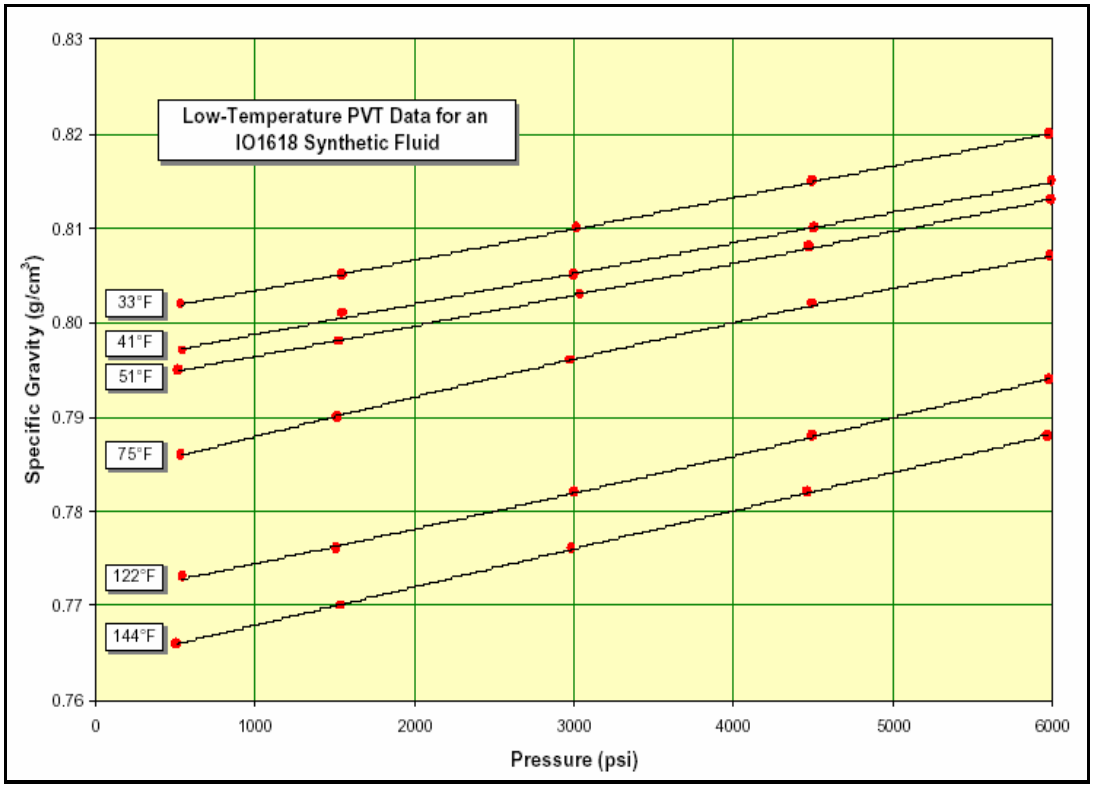
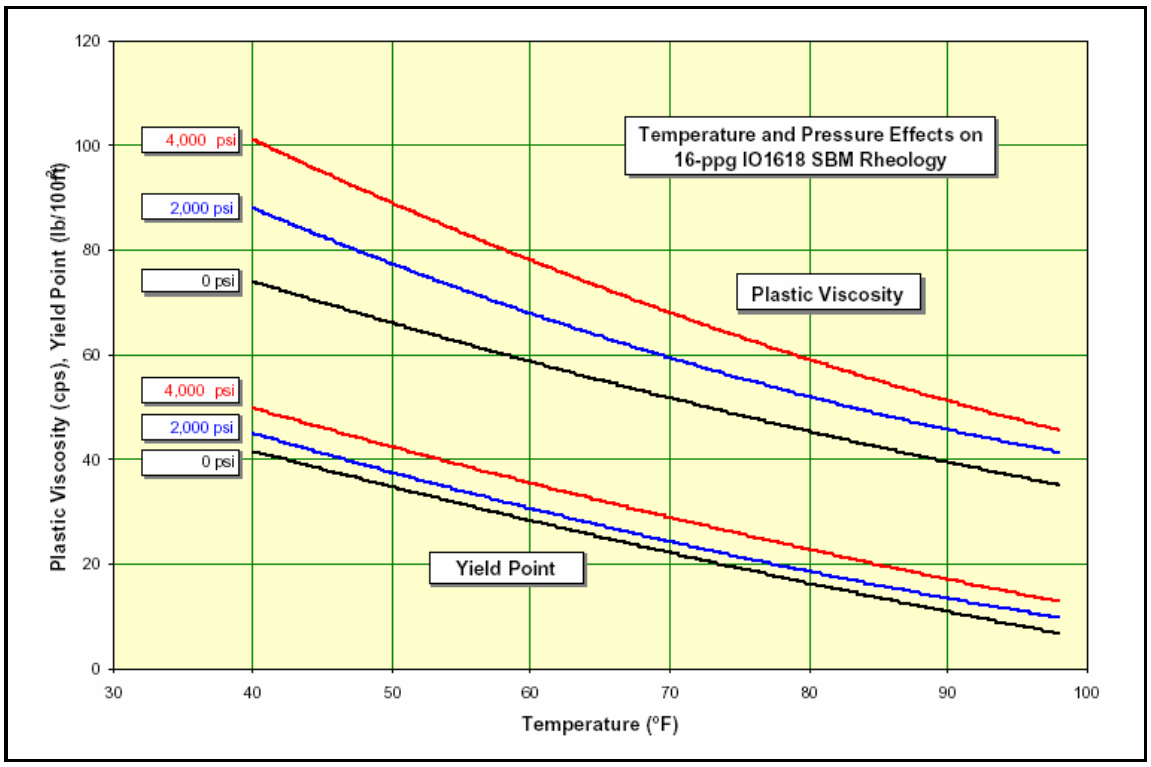



Fig. 3.7 - Low-temperature PVT data for an IO1618 synthetic fluid run on a Huxley-Bertram HTHP viscometer (from Zamora and Sanjit[17]).

**Fig. 3.8** presents temperature and pressure effects on basic rheological parameters of a 16.0-lb/gal IO1618 SBM as measured on a Fann Model 75 viscometer.




3.8 – Low-temperature and pressure effects on PV and YP of a 16-lb/gal, 85:15 oil/water ratio IO1618 synthetic mud (from Zamora and Sanjit[17]).

The impact of cold temperatures experienced in deep water is clearly demonstrated in the last two figures. One consequence is that mud weights must be associated with the temperature at which they are measured. Another is that rheology on deepwater rigs is now routinely measured at three or more different temperatures and synchronized with Fann Model 70/75 viscometer tests run periodically in the lab.[17]

# CHAPTER IV
# ACCURATE PROCEDURE FOR SELECTING THE BEST RHEOLOGICAL MODELS

Most drilling fluids used today are dispersions. Many fluid properties depend on the system's rheology. The rheology of dispersions is complex, since they usually exhibit non-Newtonian behavior. Non-Newtonian fluids are those fluids that do not conform to a direct proportionality between shear stress and shear rate, and no single equation has been proved to describe exactly the rheogram of all such fluids.

Conventional rheological models in widespread use for the past half century in the oil industry include the Bingham plastic, Power-law, and Newtonian models. Of these, the Bingham plastic is advantageous because it includes a yield point that is a positive shear stress at zero shear rate, which most drilling fluids, cement slurries, and spaces have.[18]

More recently, the Herschel-Bulkley model has seen increased usage because it accommodates the existence of a yield point (Bingham plastic) as well as the nonlinearity of the relationship of shear stress to shear rate (Power-law).[5]
This study investigated seven major non-Newtonian rheological models to get more alternatives for selecting the best model that represents accurately the shear stress-shear rate relationship for a given non-Newtonian fluid. These models are the Bingham, Power-law, API RP 13D, Herschel-Bulkley, Unified, Robertson and Stiff, and Casson. To determine which rheological model best fit the behavior of the drilling fluid, we plotted the shear stress versus shear rate data of the drilling fluid.

We assumed that the model which gives the lowest absolute average percent error ($E_{AAP}$) between the measured and calculated shear stresses is the best one for a given non-Newtonian fluid.

Selection of the best model is of great importance in achieving correct results for pressure drop and hydraulics calculations.

### 4.1 Newtonian Model

A fluid that has a constant viscosity at all shear rates at a constant temperature and pressure is called a Newtonian fluid. Also, it can be described by a one-parameter rheological model. An equation describing a Newtonian fluid is given below:

$$\tau = \mu\gamma \quad \text{.....(4.1)}$$

When the shear stress ($\tau$) of a Newtonian fluid is plotted against the shear rate ($\gamma$) in linear coordinates a straight line through the origin results. The Newtonian viscosity ($\mu$) is the slope of this line.

**Table 4.1** is an example to follow through this entire chapter.

**Table 4.1—Data from Fann 70 (from White and Zamora[9])**

| RPM | Reading |
|---|---|
| 600 | 92 |
| 300 | 58 |
| 200 | 46 |
| 100 | 32 |
| 6 | 10 |
| 3 | 8 |

To transform the laboratory data units to field engineering units (**Table 4.2**), we have to apply conversion factors:

$$\gamma = 1.703V, \quad \text{..... (4.2)}$$

$$\tau = 1.067R. \quad \text{..... (4.3)}$$

**Table 4.2—Shear Stress Measured in Field Units**

| $\gamma$ ($sec^{-1}$) | $\tau$ ($lbf/100ft^2$) |
|---|---|
| 1021.8 | 98.164 |
| 510.9 | 61.886 |
| 340.6 | 49.082 |
| 170.3 | 34.144 |
| 10.22 | 10.67 |
| 5.11 | 8.536 |

**Fig. 4.1** shows the Newtonian rheogram; from the equation of straight line we can estimate the slope, $\mu$= 0.1066 lbf.sec/100 $ft^2$. The straight line was obtained using linear regression techniques.

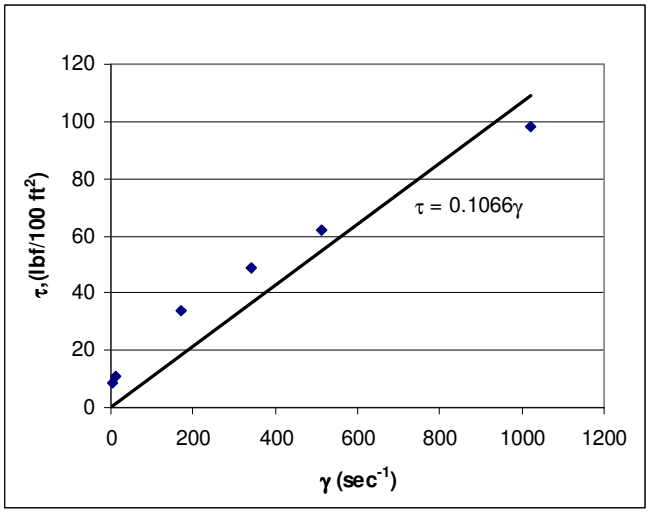



Fig. 4.1—Newtonian fluid rheogram.

To estimate viscosity in field units (cp) we have to convert by the following equation:

$$\mu = 47880m/100. \quad \text{..... (4.4)}$$

Our result is 51 cp.

Now, we can estimate the shear stresses as function of viscosity. **Table 4.3** shows the results

**Table 4.3—Shear Stress Calculated as Function of Viscosity**

| $\gamma$ ($sec^{-1}$) | $\tau$ lbf/100$ft^2$ |
|---|---|
| 1021.8 | 108.92388 |
| 510.9 | 54.46194 |
| 340.6 | 36.30796 |
| 170.3 | 18.15398 |
| 10.218 | 1.0892388 |
| 5.109 | 0.5446194 |

To estimate the $E_{AAP}$, we used a statistical method. This method is used between the measured and calculated shear stresses:

$$E_{AAP} = \left[(1/N) \sum \left| (\tau_{measured} - \tau_{calculated})/\tau_{measured} \right| \right] x100. \quad \text{.....(4.5)}$$

Using this example, for the Newtonian model $E_{AAP}$ = 46.54%. **Fig. 4.2** shows a comparison between measured and calculated data.

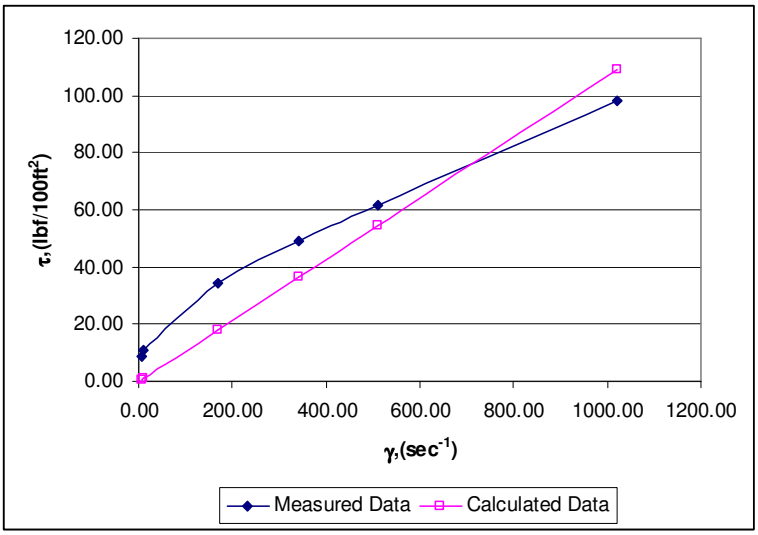



Fig. 4.2— Comparison between measured data and calculated data for Newtonian model.

Note that estimation of Newtonian viscosity can be made in an easier way by estimating the viscosity equal to the reading at 300 RPM, $R_{300}$.[14]
Then for our case, $\mu$= 58 cp. This equation is used for hydraulics calculations.

### 4.2 Bingham Plastic Model

The Bingham plastic model was the first two-parameter model that gained widespread acceptance in the drilling industry and is simple to visualize. However, it does not represent accurately the behavior of the drilling fluid at very low shear rates (in the annulus) or at very high shear rate (at the bit).[19]

$$\tau = \mu_p \gamma + \tau_y . \quad \text{..... (4.6)}$$

The Bingham parameters, yield point ($\tau_y$) and plastic viscosity ($\mu_p$) can be read from a graph or can be calculated by the following equations,[14]

$$\mu_p = R_{600} - R_{300.} \quad \text{.....(4.7)}$$

$$\tau_y = R_{300} - \mu_p . \quad \text{..... (4.8)}$$

Let us consider the same data used in the Newtonian model to show the calculations for the Bingham plastic model. **Fig. 4.3** and **Table 4.4** show the results.

The straight line was obtained using linear regression techniques.

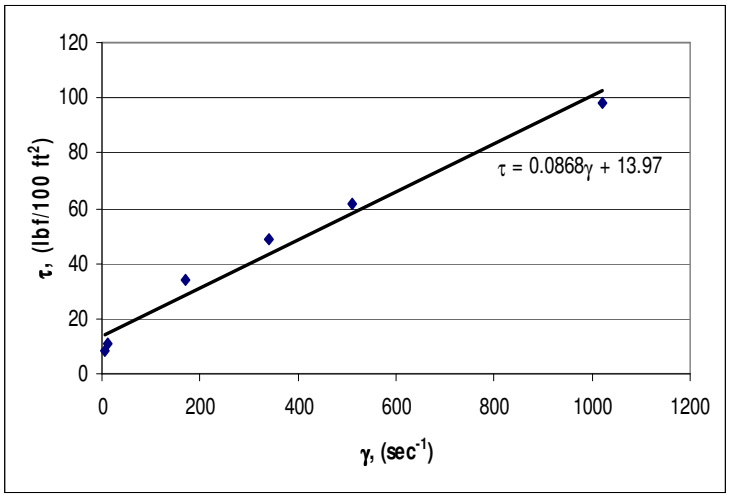



Fig. 4.3— Bingham plastic fluid rheogram.

To estimate viscosity in field units (cp), we have to convert with Eq. 4.4:

$\mu_p$ = 0.0868x47880/100=41.55 cp.

$\tau_y$ = 13.97 lbf/100 ft$^2$.

Using Eq. 4.7 and Eq. 4.8, we have[14]

$\mu_p$= 92-58= 34 cp.

$\tau_y$=58-34=24 lbf/100 ft$^2$.

Note, we are considering the graph to estimate $E_{AAP}$ and Eq. 4.7 and Eq. 4.8 for hydraulics.

**Table 4.4—Shear Stress Calculated as Function of Plastic Viscosity and Yield Point**

| $\gamma$ ($sec^{-1}$) | $\tau$ $lbf/100ft^2$ |
|---|---|
| 1021.8 | 102.635866 |
| 510.9 | 58.30296813 |
| 340.6 | 43.5253355 |
| 170.3 | 28.74770289 |
| 10.218 | 14.85672822 |
| 5.109 | 14.41339924 |

Eq. 4.5 was used to estimate the absolute average percent error ($E_{AAP}$), which for this example, for the Bingham plastic model, is 24.26%. **Fig. 4.4** shows a comparison between measured and calculated data.

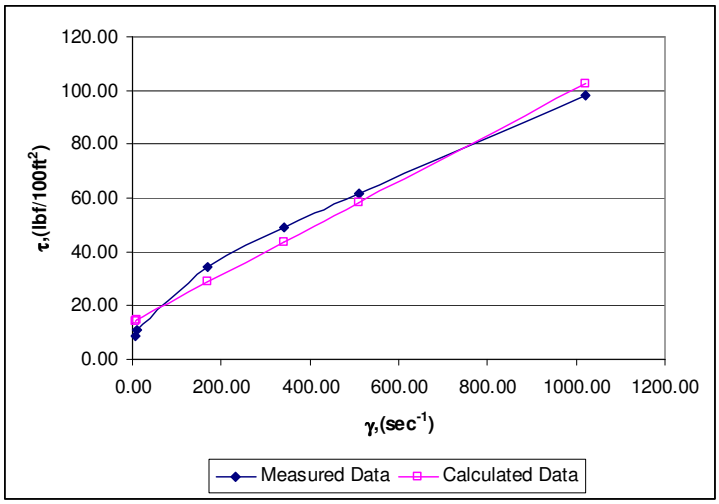



Fig. 4.4— Comparison between measured data and calculated data for Bingham plastic model.

Note that the yield strength, $\tau_0$, is the true shear stress at zero shear-rate and relates to the state of flocculation of the drilling fluid at rest. It is more representative of the structure formed at rest than the yield point value. And its value is usually approximated by measuring the shear stress at 3 RPM.

**4.3 Power Law Model**

The Bingham plastic model assumes a linear relationship between shear stress and shear rate. However, a better representation of the behavior of a drilling fluid is to consider a Power-law relationship between viscosity and shear rate such that:

$$\tau = k\dot{\gamma}^n, \quad \text{..... (4.10)}$$

where *k* is the consistence index and *n* is flow behavior index.

Eq. 4.10 was linearized as follows:

$$\log\tau = \log k + n\log\dot{\gamma}, \quad \text{..... (4.11)}$$

where *n* is determined from the slope and *k* is the intercept.

The Power-law model provides more information in the low-shear-rate condition but still has a weakness at high shear rates.[19]

Let us consider the data given in the Newtonian model to illustrate the calculations. The first step is to obtain a logarithmic graph shear rate and shear stress from Table 4.2.

**Fig. 4.5** and **Table 4.5** show the results. The straight line was obtained using linear regression techniques (least-squares regression).

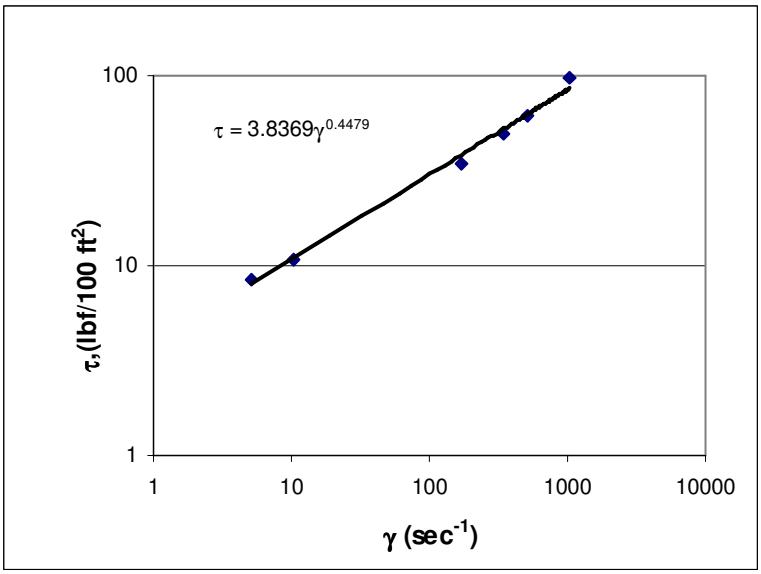



Fig. 4.5— Power-law fluid rheogram.

From **Fig. 4.5** the Power law parameters are:

$n$= 0.4479

$k$ = 3.8369 lbf.sec$^n$/100ft$^2$

**Table 4.5—Shear Stress Calculated as Function of Power Law Parameters**

| $\gamma$ (sec$^{-1}$) | $\tau$ lbf/100ft$^2$ |
|---|---|
| 1021.8 | 85.455419 |
| 510.9 | 62.65009346 |
| 340.6 | 52.24663401 |
| 170.3 | 38.30367391 |
| 10.218 | 10.86498097 |
| 5.109 | 7.965464115 |

One of the obvious disadvantages of the Power law is that it fails to describe the low-shear-rate region. Since $n$ is usually less than one, at low shear rate $\mu$ goes to infinity (only as $\gamma \rightarrow 0$) rather than to a constant, as usually observed experimentally. Viscosities also become Newtonian at high shear rates for many suspensions and dilute polymer solutions.[13]

Using Eq. 4.5, $E_{AAP}$ = 6.88%. **Fig. 4.6** shows a comparison between measured and calculated data.

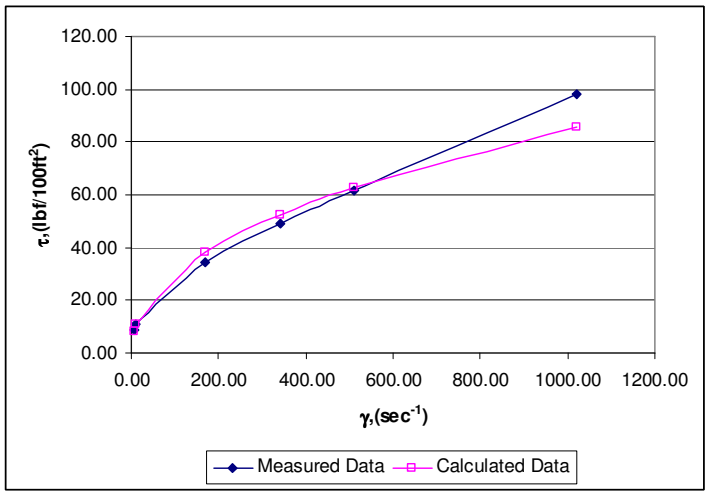



Fig. 4.6— Comparison between measured data and calculated data for Power law model.

Note that the estimations of Power-law parameters can be made by the following equations[14]:

$$n = 3.32 \log\left(\frac{R_{600}}{R_{300}}\right), \quad \text{.....(4.12)}$$

$$k = 510 R_{600}/511^{n}. \quad \text{.....(4.13)}$$

Then for our case, $n$= 0.6652 and $k$= 467.06 dyne.sec$^n$/100cm$^2$. These equations to estimate Power law parameters are used for hydraulics calculation in Chapter V.

### 4.4 API Model (RP 13D)

API published their API RP 13D[1] in 1995. In this publication, the API recommends using a modified Power-law model to calculate pressure losses in pipes and annuli. For a Power-law model, the apparent viscosity decreases with increasing shear rate (Eq. 4.10).

The API Power law tries to match shear rates from the viscometer with shear rates actually experienced inside the drillpipe and annulus. Inside the drillpipe, 600 and 300 RPM readings are used for rheology and pressure loss calculations.

- Pipe Flow

$$n_p = 3.32\log\left(\frac{R_{600}}{R_{300}}\right). \quad \text{.....} (4.14)$$

$$k_p = \frac{5.11R_{600}}{1{,}022^{n_p}}. \quad \text{.....} (4.15)$$

Inside the annulus, 3 and 100 RPM readings are used for rheology and pressure- loss calculations.

- Annulus Flow:

$$n_a = 0.657\log\left(\frac{R_{100}}{R_3}\right). \quad \text{.....} (4.16)$$

$$k_a = \frac{5.11R_{100}}{170.2^{n_a}}. \quad \text{.....} (4.17)$$

As shown, RP 13D is based on a "dual Power law," the lower shear rate segment for the annulus and the upper segment for inside the drillstring. Fig. 4.7 and Table 4.6 show the results. The straight lines were obtained using linear regression techniques.

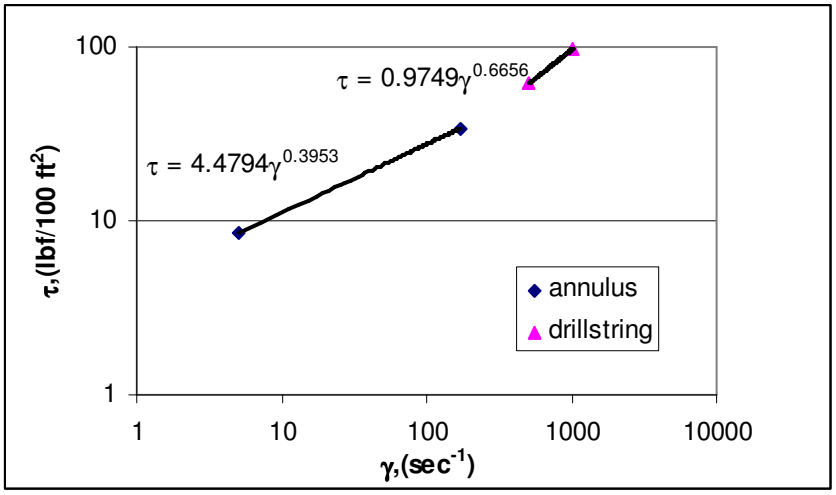



Fig. 4.7— API "dual power law" fluid rheogram.

From **Fig. 4.7** the API parameters are:

$n_p$= 0.6656.

$n_a$=0.3953.

$k_p$ = 0.9749 lbf.sec$^n$/100ft$^2$.

$k_a$ = 4.4794 lbf.sec$^n$/100ft$^2$.

From Eq. 4.14 to Eq. 4.17,

$n_p$= 3.32log(92/58)=0.66519465

$n_a$= 0.657log(32/8)=0.395553

$k_p$ = 5.11(92/1022$^{0.6652}$)=4.6808 dyne.sec$^n$/cm$^2$

$k_a$ = 5.11(32/170.2$^{0.3956}$)=21.4291 dyne.sec$^n$/cm$^2$

**Table 4.6—Shear Stress Calculated as Function of API Parameters**

| $\gamma$ ($sec^{-1}$) | $\tau$ $lbf/100ft^2$ |
|---|---|
| 1021.8 | 98.17344076 |
| 510.9 | 61.89113501 |
| 340.6 | 47.2522078 |
| 170.3 | 34.17388941 |
| 10.218 | 11.22874048 |
| 5.109 | 8.535787172 |

Using Eq. 4.5, $E_{AAP}$ = 1.51%. **Fig. 4.8** shows a comparison between measured and calculated data.

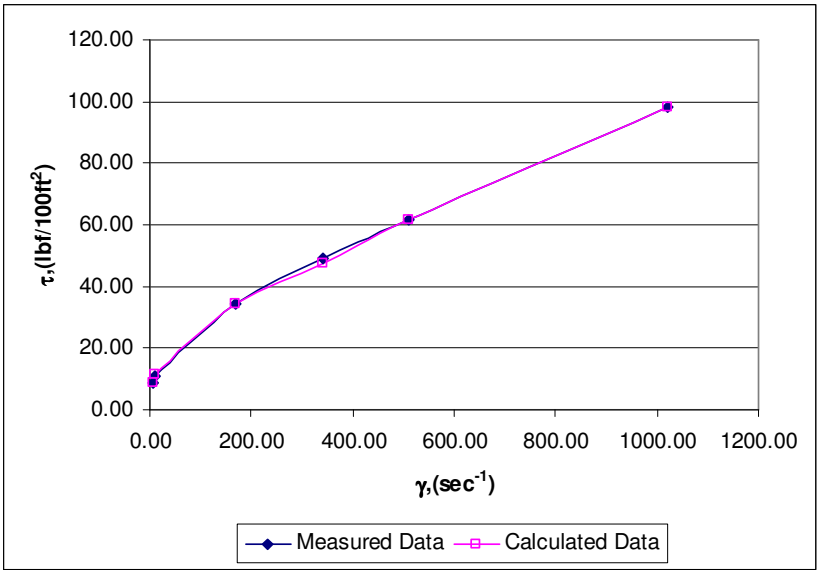



Fig. 4.8— Comparison between measured data and calculated data for API model.

This is a good choice for pressure loss calculations. The approach technically is a “generalized correlation” for which explicit laminar flow solutions are both available and straightforward. However, this approach does not consider a yield stress term that has become central to evaluating and optimizing hole cleaning, barite sag, suspension, and other key drilling concerns.[18]

### 4.5 Herschel-Bulkley

The Herschel-Bulkley model defines a fluid by three-parameter and can be described mathematically as follows:

$$\tau=\tau_0+k\gamma^n. \quad \text{.....(4.18)}$$

$$\log(\tau-\tau_0)=\log(k)+n\log(\gamma). \quad \text{.....(4.19)}$$

For $\tau < \tau_0$ the material remains rigid. For $\tau > \tau_0$, the material flows as a Power-law fluid.

The Herschel-Bulkley equation is preferred to Power-law or Bingham relationships because it results in more accurate models of rheological behavior when adequate experimental data are available. The yield stress is normally taken as the 3 RPM reading. However, we are taking Versan and Tolga’s[20] approach to obtain $\tau_0$. Then *n* and *k* values can be calculated from the 600 and 300 RPM values or graphically. The Power-law model described above is valid for fluids for which the shear stress is zero when the strain rate is zero.

The Herschel-Bulkley model is commonly used to describe materials such as concrete, mud, dough, and toothpaste, for which a constant viscosity after a critical shear stress is a reasonable assumption when a log-log graph is made. In addition to the transition behavior between a flow and no-flow regime, the Herschel-Bulkley model can also exhibit a shear-thinning or shear thickening behavior depending on the value of *n*.

Since this is a three-parameter model, an initial calculation of $\tau_0$ is required for other parameter calculations. $\tau_0$ is calculated by Versan and Tolga[20].

$$\tau_0 = \frac{\tau^{*2} - \tau_{min} \times \tau_{max}}{2 \times \tau^{*} - \tau_{min} - \tau_{max}}, \quad \text{.....(4.20)}$$

where $\tau^*$ is the shear stress value corresponding to the geometric mean of the shear rate, $\gamma^*$.

$$\gamma^{*} = \sqrt{\gamma_{min} \gamma_{max}} \, . \quad \text{.....(4.21)}$$

From Eq. 4.21, $\gamma^*$ = 72.25 $sec^{-1}$. Then using this value we can interpolate between values of shear stress in Table 4.2,
$\tau^*$=19.77 lbf/100$ft^2$.

Finally, from Eq. 4.20: $\tau_0$ = 6.66 lbf/100$ft^2$.

**Fig. 4.9** and **Table 4.7** show the results. The straight line was obtained using linear regression techniques.

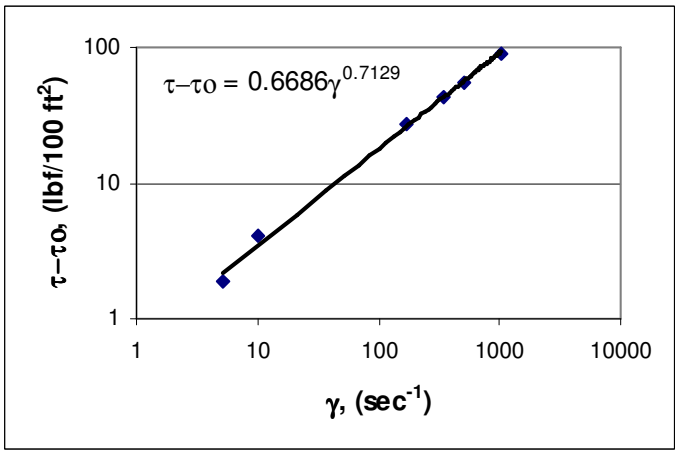



Fig. 4.9— Herschel-Bulkley fluid rheogram.

From **Fig. 4.9** the Herschel-Bulkley parameters are:

$n$= 0.7129.

$k$ = 0.6686 lbf.sec$^n$/100ft$^2$

**Table 4.7—Shear Stress Calculated as Function of Herschel-Bulkley Parameters**

| $\gamma$ (sec$^{-1}$) | $\tau$ lbf/100ft$^2$ |
|---|---|
| 1021.8 | 100.0862 |
| 510.9 | 63.6589 |
| 340.6 | 49.3504 |
| 170.3 | 32.7048 |
| 10.218 | 10.1635 |
| 5.109 | 8.7968 |

Using Eq. 4.5, $E_{AAP}$ = 2.90%. **Fig. 4.10** shows a comparison between measured and calculated data.

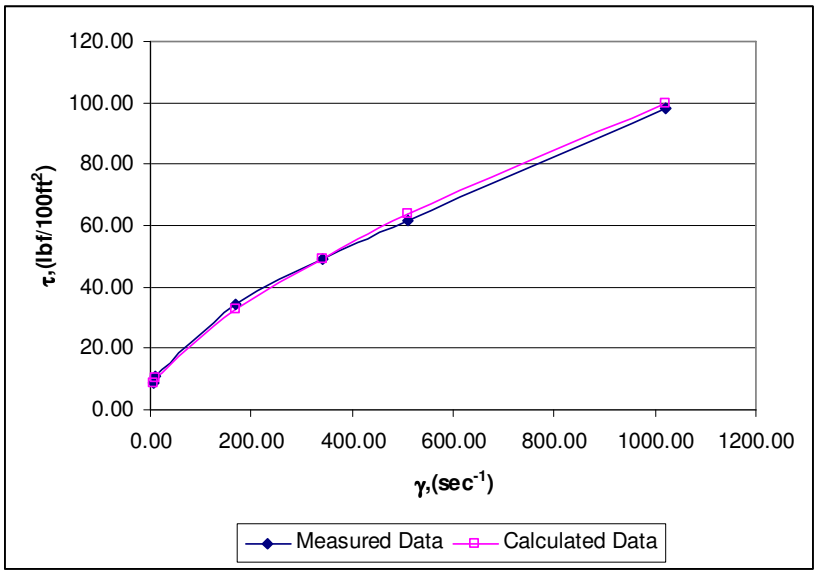



Fig. 4.10— Comparison between measured data and calculated data for Herschel-Bulkley model.

### 4.6 Unified Model

The Unified model[2] is an improved version of a simplified Herschel-Bulkley model established by the drilling industry years ago. See Eq. 4.18 and Eq. 4.19. The calculations of rheological parameters for the Unified model *n* and *k* involve previous estimation of plastic viscosity *($\mu_p$),* yield point *($\tau_y$),* and yield stress *($\tau_o$).*

See Eq. 4.7 and Eq. 4.8 for estimation of plastic viscosity and yield point respectively.

To estimate $\tau_0$ for the Unified model, Zamora and Power[2] give the following alternative: Take low shear yield point ($\tau_{yL}$) as $\tau_0$. This is calculated from Eq. 4.22.

$$\tau_{yL} = (2R_3 - R_6)1.066, \quad \text{.....(4.22)}$$

where $\tau_{yL}$ is lower shear yield point.

For the example that we have been following:

$\tau_{yL}$ = (2x8-10) x1.066= 6.396 lbf/100ft$^2$.

The equations proposed for this model to estimate $n_p$ and $n_a$, and $k_p$ and $k_a$ are the following:

- Pipe Flow

$$n_p = 3.32\log\left(\frac{2\mu_p + \tau_y}{\mu_p + \tau_y}\right). \quad \text{.....(4.23)}$$

$$k_p = 1.066\left(\frac{\mu_p + \tau_y}{511^{n_p}}\right). \quad \text{.....(4.24)}$$

- Annular Flow

$$n_a = 3.32\log\left(\frac{2\mu_p + \tau_y - \tau_o}{\mu_p + \tau_y - \tau_0}\right) \quad \text{.....(4.25)}$$

$$k_a = 1.066\left(\frac{\mu_p + \tau_y - \tau_0}{511^{n_a}}\right) \quad \text{.....(4.26)}$$

Now, let estimate $n$ and $k$ for the example that we have been following:

- Pipe Flow

$$n_p = 3.32\log\left(\frac{2\times 34 + 24}{34 + 24}\right) = 0.665.$$

$$k_p = 1.066\left(\frac{34 + 24}{511^{0.665}}\right) = 0.971 \text{ lbf.sec}^n/100\text{ft}^2.$$

- Annular Flow

$$n_a = 3.32\log\left(\frac{2\times 34 + 24 - 6.396}{34 + 24 - 6.396}\right) = 0.73.$$

$$k_a = 1.066\left(\frac{34 + 24 - 6.396}{511^{0.73}}\right) = 0.577 \text{ lbf.sec}^n/100\text{ft}^2.$$

The ratio $\tau_0/\tau_y$ is another parameter which is a useful tool to help characterize fluids rheologically, although it is not necessary for solving the model.Some fluids may exhibit more plastic behavior in one part of the well and more pseudoplastic behavior in another. This is important for hole cleaning and barite sag considerations.

As the ratio $\tau_0/\tau_y$ approaches 1, $(\tau_0 \rightarrow \tau_y)$, fluids take on Bingham plastic behavior. For $\tau_0/\tau_y$ approaching 0, $(\tau_0 \rightarrow 0)$, they behave more like pseudoplastic (Power-law) fluids.

For our example:

$\tau_0/\tau_y$ =6.4/24= 0.27

Clearly, the fluid behaves more like a pseudoplastic.

**Table 4.8** shows the result of shear stress calculation using the parameter estimated above.

**Table 4.8—Shear Stress Calculated as Function of Unified Model Parameters**

| $\gamma$ ($sec^{-1}$) | $\tau$ $lbf/100ft^2$ |
|---|---|
| 1021.8 | 97.48 |
| 510.9 | 61.47 |
| 340.6 | 46.94 |
| 170.3 | 30.93 |
| 10.218 | 9.55 |
| 5.109 | 8.30 |

Using Eq. 4.5, $E_{AAP}$=4.74%. **Fig. 4.11** shows a comparison between measured and calculated data.

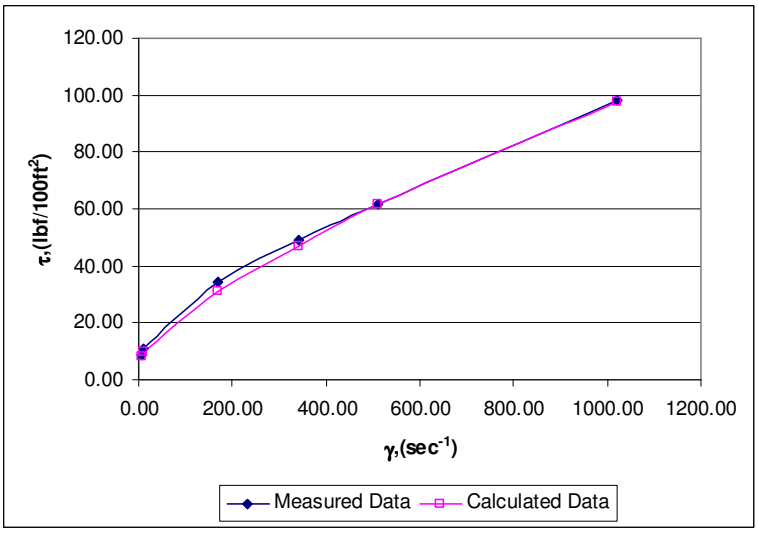



Fig. 4.11— Comparison between measured data and calculated data for Unified model.

### 4.7 Robertson and Stiff Model

Robertson and Stiff[21] developed a more general model to describe the rheological behavior of drilling fluids and cement slurries. The basic equation is:

$$\tau = A\,(\dot{\gamma}+C)^{B}, \quad \text{.....(4.27)}$$

where *A, B, and C* are model parameters. *A* and *B* can be considered similar to the parameters *k* and *n* of the Power-law model. The third parameter *C* is a correction factor to the shear rate, and the term $(\dot{\gamma}+C)$ is considered effective shear rate.

Eq. 4.28 represents the yield stress for the Robertson and Stiff model.

$$\tau_0 = AC^{B}. \quad \text{.....(4.28)}$$

Despite the fact that some investigators[22] have meticulously shown that the Robertson and Stiff model is superior to Bingham and Power-law models, it has

found little relevance in the drilling industry because of the relative complexity in evaluating the three parameters, *A, B* and *C*.

The major advantage of the model over the Power-law and Bingham plastic models is the superior fit of rheological stress/rate of strain data.[21]

To evaluate the parameters,[22] we plotted the shear stress corresponding to several shear rates. The logarithm from Eq. 4.27 plots a straight line on log-log coordinates:

$$\log(\tau) = \log(A) + B \log(\dot{\gamma} + C). \quad \text{.....(4.29)}$$

Thus, if $\tau$ is plotted vs. $(\gamma + C)$ on log-log coordinates, $B$ is the slope and $A$ is the intercept where $(\gamma + C) = 1.0$.

$$C = (\gamma_{min}\gamma_{max} - \gamma^{*2}) / (2\gamma^{*} - \gamma_{min} - \gamma_{max}), \quad \text{.....(4.30)}$$

where $\gamma^{*}$ is the shear rate value corresponding to the geometric mean of the shear stress, $\tau^{*}$.

The geometric mean of the shear stress ($\tau^{*}$) is then calculated from:

$$\tau^{*} = (\tau_{min} \times \tau_{max})^{1/2}. \quad \text{.....(4.30)}$$

From Eq. 4.30, $\tau^{*}$= 28.95 lbf/100ft$^2$. Then with this value we can interpolate between the values of shear rates in **Table 4.2**, $\gamma^{*}$=134.86 1/sec$^{B}$.

Finally, from Eq. 4.30, C = 17.12 1/sec$^{B}$.

The **Fig. 4.12** shows the results. The straight line was obtained by using linear regression techniques.

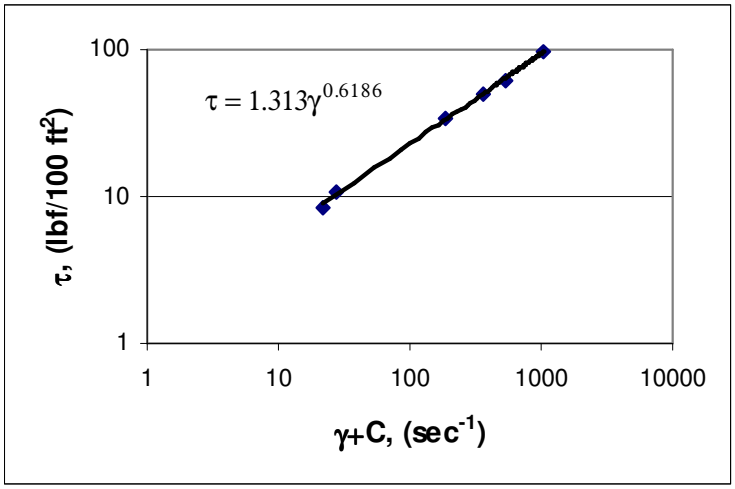



Fig. 4.12— Robertson and Stiff fluid rheogram.

From Fig. 4.12 the Robertson and Stiff parameters are:

$A$ = 1.31297551 lbf.sec$^{B}$/100 ft $^{2}$.

$B$= 0.618576471.

**Table 4.9** shows the result of shear-stress calculation using the parameter estimated above.

**Table 4.9—Shear Stress Calculated as Function of Robertson and Stiff Parameters**

| $\gamma$ (sec$^{-1}$) | $\tau$ lbf/100ft$^2$ |
|---|---|
| 1021.8 | 96.4372 |
| 510.9 | 63.4492 |
| 340.6 | 49.8680 |
| 170.3 | 33.4329 |
| 10.218 | 10.1637 |
| 5.109 | 8.9430 |

Using Eq. 4.5, $E_{AAP}$=2.9137%. **Fig. 4.13** shows a comparison between measured and calculated data.

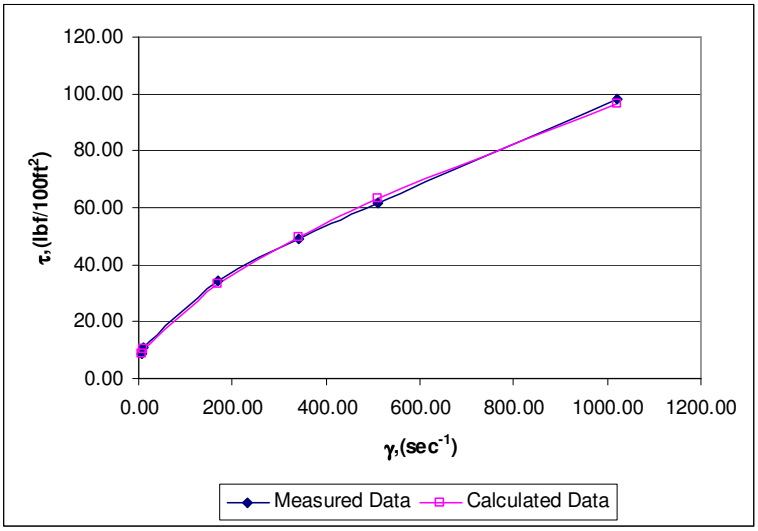



Fig. 4.13— Comparison between measured data and calculated data for Robertson and Stiff model.

### 4.8 Casson Model

Casson's 1959 model described the flow of viscoelastic fluids. This model has a more gradual transition from Newtonian to the yield region. For many materials, such as blood and food products, it provides a better fit. Note that values of the parameters for the Casson model also depend on the range of shear rates considered.[13]

This model is used by petroleum engineers in the characterization of cement slurry and is better for predicting high shear-rate viscosities when only low and intermediate shear-rate data are available. The Casson model is more accurate at both very high and very low shear rate.[19]

The Casson model has been used in other industries to give a more accurate representation of high shear rate viscosities when only low and intermediate shear-rate data are available. Thus, this model will improve our ability to predict viscosities at the bit.[19]

Casson considered rigid primary particles aggregating into long rods. Under shear, the rod length progressively decreases until at very high shear rate, the rod is completely broken down into primary particles.[19]

The empirical equation for the 1D form of the Casson model is given by[13]

$$\gamma = 0 \qquad \text{For } \tau < \tau_c. \quad \text{.....(4.31)}$$

$$\tau^{\frac{1}{2}} = \tau_c^{\frac{1}{2}} + \mu_c^{\frac{1}{2}}\gamma^{\frac{1}{2}} \qquad \text{For } \tau \geq \tau_c, \quad \text{.....(4.32)}$$

where $\tau_c$ is the Casson yield stress and $\mu_c$ is the Casson plastic viscosity.

**Table 4.10** shows the values of shear rates and shear stresses needed to build **Fig. 4.14.**

**Table 4.10—Square Roots of Variables Used to Graph Fig. 4.14**

| $\gamma$ ($sec^{-1}$) | $\tau$ ($lbf/100ft^2$) |
|---|---|
| 1021.8 | 101.1646 |
| 510.9 | 61.7832 |
| 340.6 | 47.4246 |
| 170.3 | 31.5577 |
| 10.218 | 10.9202 |
| 5.109 | 9.5159 |

The straight line in **Fig. 4.14** was obtained by linear regression techniques.

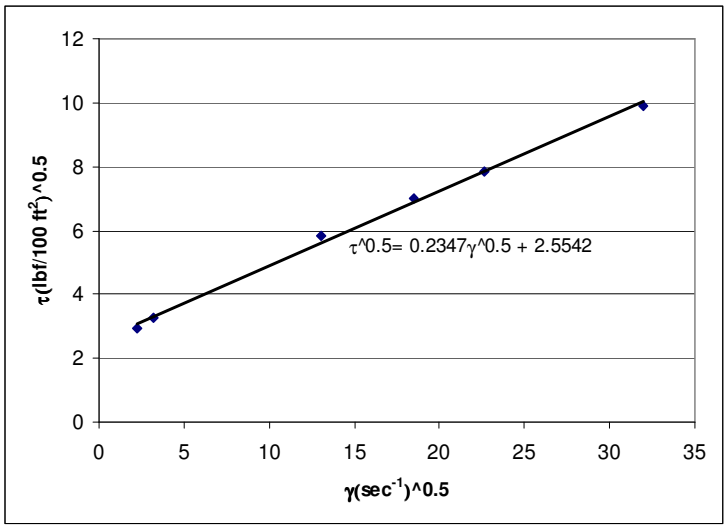



Fig. 4.14— Casson fluid rheogram.

Then we can obtain from **Fig. 4.14** the Casson model parameters:

$\tau_c^{0.5}$ = 2.554 lbf/100 ft$^2$,

$\mu_c^{0.5}$ = 0.2347 lbf.sec/100 ft$^2$,

or

$\tau_c$ = 6.5238 lbf/100 ft$^2$.

and

$\mu_c$ = 0.0551 lbf.sec/100 ft$^2$.

**Table 4.11** shows the result of shear-stress calculation using the parameters estimated above.

**Table 4.11—Shear Stress Calculated as Function of Casson Model Parameters**

| $\gamma$ ($sec^{-1}$) | $\tau$ lbf/100$ft^2$ |
|---|---|
| 1021.8 | 101.1646 |
| 510.9 | 61.7832 |
| 340.6 | 47.4246 |
| 170.3 | 31.5577 |
| 10.218 | 10.9202 |
| 5.109 | 9.5159 |

Using Eq. 4.5, $E_{AAP}$ = 4.66%. **Fig. 4.15** shows a comparison between measured and calculated data.

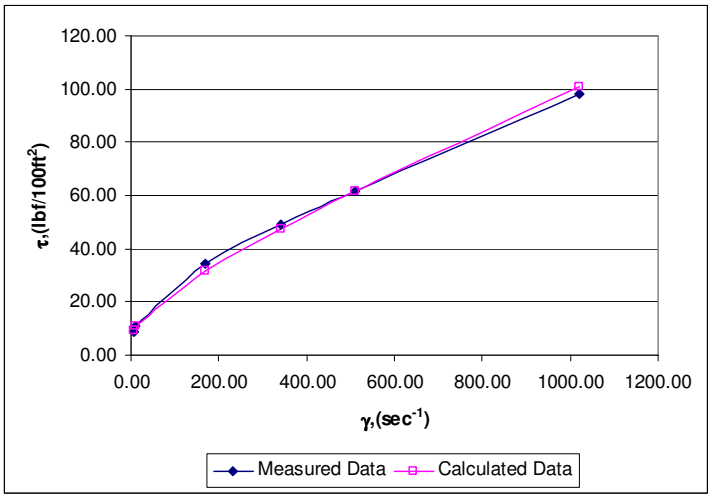



Fig. 4.15— Comparison between measured data and calculated data for Casson model.

# CHAPTER V
# HYDRAULICS

Conventional calculations of downhole pressure, which assume constant drilling fluid properties, are both practical and accurate enough for routine wells. Downhole static pressures are easy to calculate from mud weight measured at the surface, while additional pressures caused by circulation can be calculated using established relationships between pump rate and drilling fluid rheological properties.[23]

Errors that result from ignoring variations in mud properties are small in relatively shallow wells. In these settings, mud engineers can concentrate on formulating drilling fluid properties for maximum rates of penetration and optimal hole conditions. Formations can commonly withstand moderate overpressure before being fractured, which permits mud engineers to add a comfortable safety margin when weighting the mud.[23]

On the other hand, in high pressure and high temperature (HPHT), extended reach, and deepwater wells as established before, mud properties do vary with downhole pressure and temperature, affecting the accuracy of both surface measurements and downhole estimations of mud weight and viscosity. In these wells these variations can be significant because of the limited safety margins available.[23]

Clearly the ability to predict these effects is critical to the successful drilling of HPHT, extended reach, and deepwater wells. Small but serious errors in computing the drilling fluid pressure at the reservoir may result from ignoring uncertainties in either temperature or fluid properties. Simulation of downhole

temperature profiles at all phases of the drilling operation is therefore the key to understanding the behavior of drilling fluids.[24]

Equivalent circulating density (ECD) is often much higher than equivalent mud weight (EMW) in HPHT, extended reach, and deepwater wells due to the small annular clearances between the drillpipe and hole wall. ECD is computed from the dimensions of the annulus and, for a given fluid viscosity, increases with pump rate. The calculation becomes increasingly complicated when changes of viscosity with temperature are considered.[23]

### 5.1 Frictional Pressure Loss Calculation

During circulating of drilling fluid, friction between the drilling fluid and the wall of the drill pipe and annulus cause pressure loss.[14] Actually, the pump pressure, $\Delta p_p$, is affected by:

1. Frictional pressure losses ($\Delta p_s$) in the surface equipment such as Kelly, swivel, standpipe.
2. Frictional pressure losses ($\Delta p_{ds}$) inside the drillstring (drillpipe, $\Delta p_{dp}$ and drill collar, $\Delta p_{dc}$).
3. Frictional pressure losses across the bit, $\Delta p_b$.
4. Frictional pressure losses in the annulus around the drillstring, $\Delta p_a$.

The mathematical expression for this is as given:

$$\Delta p_p = \Delta p_s + \Delta p_{ds} + \Delta p_b + \Delta p_a. \quad \text{.....(5.1)}$$

Error in $\Delta p_p$ is a combination of errors in the four elements. In general, frictional pressure losses across the bit, $\Delta p_b$, and the surface pipe system can be evaluated fairly accurately.

Error in $\Delta p_p$ consists primarily of errors from friction pressure losses in the drillstring and annulus. The drillstring pressure losses represent the largest component of error in the pump pressure.

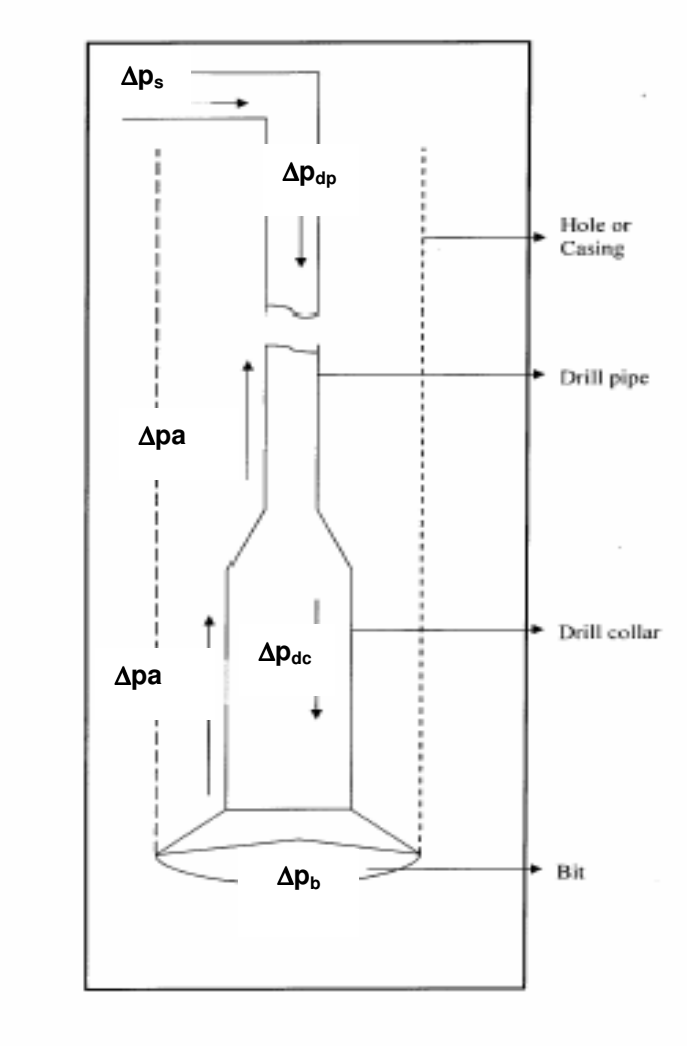



Fig. 5.1— Diagram of the drilling fluid circulating system (from Mojisola[25]).

Frictional pressure loss is a function of several factors such as rheology behavior of the drilling fluid (Newtonian or non-Newtonian), flow regime of the drilling fluid (laminar, turbulent, or intermediate flow), drilling fluid properties

(density and viscosity), flow rate of the drilling fluid ($q$), drillstring configuration and wellbore geometry. See **Fig. 5.1**

When the best-fit rheological model has been chosen and the fluid rheological properties have been determined as shown in Chapter IV, the flow regime can then be determined by calculating the Reynolds number ($N_{Re}$) at a particular fluid flow rate using the appropriate equations.

The calculated value of $N_{Re}$ is compared to a critical value $N_{Rec}$ to decide if the flow is laminar or turbulent. The next step is to calculate the friction factor, $f$. This factor is a function of the fluid rheological properties, pipe roughness, and the Reynolds number for some model.

Once the friction factor has been determined, the frictional pressure loss can be calculated using the appropriate equation from each rheological model. This chapter shows how this procedure works with each rheological model.

Appendix A shows the rheological and hydraulic equations for eight models. Appendix B also shows a numerical example for each rheological model to illustrate the pump pressure calculation.

### 5.1.1 Frictional Pressure Loss Calculation for Newtonian Fluid

- Pipe flow[14]

a. Pipe velocity:

$$v_p = \frac{0.408q}{D_p^{\,2}}. \quad \text{.....(5.2)}$$

b. Reynolds number:

$$N_{Re} = \frac{928 D_p v_p \rho}{\mu_a}. \quad \text{.....(5.3)}$$

c. Critical Reynolds number value, $N_{Rec} = 2100$.

d. Regime flow determination,

Comparison between $N_{Re}$ and $N_{Rec}$

If $N_{Re} < N_{Rec} \rightarrow$ flow is laminar.

$f = 16 / N_{Re}$. .....(5.4)

If $N_{Re} > N_{Rec} \rightarrow$ flow is turbulent.

$f = 0.0791 / N_{Re}^{0.25}$. .....(5.5)

e. Frictional pressure loss calculation inside drillstring:

$$\left(\frac{dp}{dL}\right) = \frac{f v_p^{\,2} \rho}{25.81 D_p}. \quad \text{.....(5.6)}$$

$$\Delta p_{ds} = \left(\frac{dp}{dL}\right) \Delta L, \quad \text{.....(5.7)}$$

where *(dp/dL)* is the pressure gradient, psi/ft.

- Annular Flow

a. Annular velocity:

$$v_a = \frac{0.408 q}{(D_2^{\,2} - D_1^2)}. \quad \text{.....(5.8)}$$

b. Reynolds number:

$$N_{Re} = \frac{757 (D_2 - D_1) v_a \rho}{\mu_a}. \quad \text{.....(5.9)}$$

c. Critical Reynolds Number value, $N_{Rec} = 2100$.

d. Regime flow determination:

Comparison between $N_{Re}$ and $N_{Rec}$

If $N_{Re} < N_{Rec} \rightarrow$ flow is laminar.

$f = 16 / N_{Re}$. .....(5.4)

If $N_{Re} > N_{Rec} \rightarrow$ flow is turbulent.

$f = 0.0791 / N_{Re}^{0.25}$. .....(5.5)

e. Frictional pressure loss calculation in the annulus:

$$\left(\frac{dp}{dL}\right)=\frac{fv_a{}^2\rho}{25.81(D_2-D_1)}. \quad \text{.....(5.10)}$$

$$\Delta p_a=\left(\frac{dp}{dL}\right)\Delta L. \quad \text{.....(5.11)}$$

- Frictional pressure losses across the bit, $\Delta p_b$:

$$\Delta p_b=\frac{156\rho q^2}{(D_{N1}{}^2+D_{N2}{}^2+D_{N3}{}^2)^2}, \quad \text{.....(5.12)}$$

where $D_{N1}$, $D_{N2}$, $D_{N3}$ are diameters of the three nozzles.

### 5.1.2 Frictional Pressure Loss Calculation for Bingham Plastic Fluid

To calculate velocity, Reynolds numbers, critical Reynolds number value, regime flow, and frictional pressure losses, follow the procedure outlined in Section 5.1.1 (annulus and pipe).[14] Note: Use the apparent viscosity estimate for this model from Eqs. 5.13 and 5.14. Use Eqs. 4.7 and 4.8 to estimate plastic viscosity and yield point.

- Pipe Flow

$$\mu_a=\mu_p+\frac{5\tau_y D_p}{v_p}. \quad \text{.....(5.13)}$$

- Annular Flow

$$\mu_a=\mu_p+\frac{5\tau_y(D_2-D_1)}{v_a}. \quad \text{.....(5.14)}$$

Another way to determine the flow is using the Hedstrom number, $N_{He}$, to estimate the critical Reynolds number from **Fig. 5.2.** Also, we have to work with plastic viscosity to calculate the Reynolds number. Finally, we have to compare Eqs. 5.14 and 5.15.

$$N_{He} = \frac{37100\rho\tau_y D_p^{\ 2}}{\mu_p^{\ 2}}. \quad \text{.....(5.15)}$$

$$N_{\text{Re}} = \frac{928 D_p v_p \rho}{\mu_p}. \quad \text{.....(5.16)}$$

If $N_{\text{Re}} < N_{\text{Rec}}$ →flow is laminar.

If $N_{\text{Re}} > N_{\text{Rec}}$ → flow is turbulent.

Note that we also can use this second technique with the annulus.

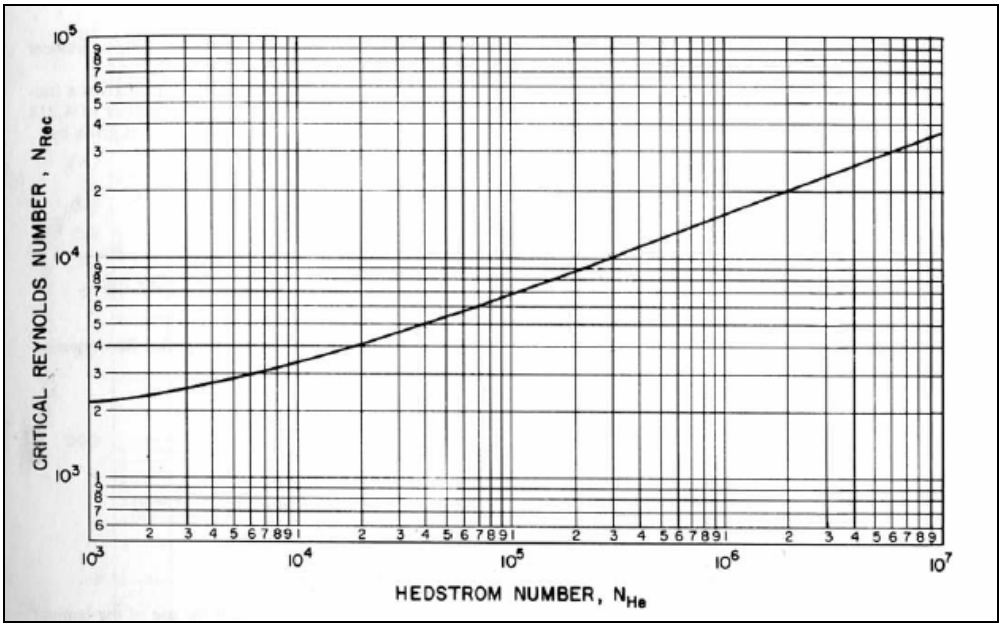



Fig. 5.2— Critical Reynolds numbers for Bingham plastic fluids (from Bourgoyne[14]).

### 5.1.3 Frictional Pressure Calculation for Power Law Fluid

To estimate velocity, we follow the procedure outlined in Part *a,* Section 5.1.1 (annulus and pipe). [14]

- Pipe Flow

b. Reynolds Number:

$$N_{\mathrm{Re}} = \frac{89100 v_p^{\,2-n} \rho}{k} \left( \frac{0.0416 D_p}{3 + \frac{1}{n}} \right)^n . \quad \text{.....(5.17)}$$

$$n = 3.32 \log\left( \frac{R_{600}}{R_{300}} \right). \quad \text{.....(4.12)}$$

$$k = \frac{510 R_{300}}{511^n} . \quad \text{.....(4.13)}$$

c. For laminar flow,[26] critical Reynolds number value, $N_{\mathrm{Re}c} = 3470\text{-}1370n$.

For turbulent flow,[26] critical Reynolds number value $N_{\mathrm{Re}c} = 4270\text{-}1370n$.

d. Regime flow determination:

Comparison between $N_{\mathrm{Re}}$ and $N_{\mathrm{Re}c}$

If $N_{\mathrm{Re}} < N_{\mathrm{Re}c} \rightarrow$flow is laminar.

The friction factor is included in Eq. 5.21.

If $N_{\mathrm{Re}} > N_{\mathrm{Re}c} \rightarrow$ flow is turbulent.[26]

$$f = \frac{a}{N_{\mathrm{Re}}{}^{b}} . \quad \text{.....(5.18)}$$

$$a = \frac{\log n + 3.93}{50} . \quad \text{.....(5.19)}$$

$$b = \frac{1.75 - \log n}{7} . \quad \text{.....(5.20)}$$

e. Frictional pressure loss calculation inside drillstring:

*Laminar:*

$$\left(\frac{dp}{dL}\right)=\frac{kv_p^n\left(\frac{3+1/n}{0.0416}\right)^n}{144000D_p^{1+n}}. \quad \text{.....(5.21)}$$

*Turbulent:*

Use Eq. 5.6 to estimate pressure loss calculation for turbulent flow.

$$\Delta p_{ds}=\left(\frac{dp}{dL}\right)\Delta L. \quad \text{.....(5.7)}$$

- Annular Flow

b. Reynolds number:

$$N_{\text{Re}}=\frac{109000v_a^{2-n}\rho}{k}\left(\frac{0.0208(D_2-D_1)}{2+\frac{1}{n}}\right)^n. \quad \text{.....(5.22)}$$

c. For laminar flow, the critical value $N_{\text{Re}c}$ = 3470-1370$n$.

For turbulent flow, the critical value $N_{\text{Re}c}$ = 4270-1370$n$.

d. Regime flow determination:

Comparison between $N_{\text{Re}}$ and $N_{\text{Re}c}$

If $N_{\text{Re}} < N_{\text{Re}c}$ →flow is laminar.

Friction factor included in Eq.5.23

If $N_{\text{Re}} > N_{\text{Re}c}$ →flow is turbulent.

Follow the same procedure as for pipe flow.

e. Frictional pressure loss calculation inside annulus*:*

*Laminar:*

$$\left(\frac{dp}{dL}\right)=\frac{kv_a^n\left(\frac{2+1/n}{0.0208}\right)^n}{144000(D_2-D_1)^{1+n}}. \quad \text{.....(5.23)}$$

*Turbulent:*

Use Eq.5.10 to estimate pressure loss calculation for turbulent flow.

$$\Delta p_a=\left(\frac{dp}{dL}\right)\Delta L. \quad \text{.....(5.7)}$$

### 5.1.4 Frictional Pressure Loss Calculation for API RP 13D Fluid

To estimate velocity, Reynolds numbers, and friction pressure losses, follow the procedure outlined in Parts *a, b and e* of Section 5.1.1 (annulus and pipe).[1] Note: Use the equivalent viscosity to estimate Reynolds number; see Eq. 5.25.

- Pipe Flow

b. Reynolds number:

$$N_{\text{Re}}=\frac{928v_p\rho D_p}{\mu_e}. \quad \text{.....(5.24)}$$

$$\mu_e=100k\left(\frac{96v_p}{D_p}\right)^{n-1}\left(\frac{3n+1}{4n}\right)^n. \quad \text{.....(5.25)}$$

$$n=3.32\log\left(\frac{R_{600}}{R_{300}}\right). \quad \text{.....(4.12)}$$

$$k=\frac{5.10R_{600}}{1022^n}, \quad \text{.....(4.15)}$$

where $\mu_e$ is the equivalent viscosity, cp

c. Critical value $N_{\text{Re}c}$ = 2100.

d. Regime flow determination:

Comparison between $N_{Re}$ and $N_{Rec}$

If $N_{Re} < N_{Rec}$ →flow is laminar.

Use Eq. 5.4.

If $N_{Re} > N_{Rec}$ →flow is turbulent.

Use Eqs. 5.18-5.20.

- Annular Flow

b. Reynolds number:

$$N_{Re} = \frac{928 v_a \rho (D_2 - D_1)}{\mu_e}. \quad \text{.....(5.26)}$$

$$\mu_e = 100k\left(\frac{144v}{D_2 - D_1}\right)^{n-1}\left(\frac{2n+1}{3n}\right)^{n}. \quad \text{.....(5.27)}$$

$$n = 0.657\log\left(\frac{R_{100}}{R_3}\right). \quad \text{.....(4.16)}$$

$$k = \frac{5.10R_{100}}{170.2^{n}}. \quad \text{.....(4.17)}$$

c. Critical value $N_{Rec} = 2100$.

d. Regime flow determination:

Comparison between $N_{Re}$ and $N_{Rec}$

Use the same procedure followed for this model in pipe flow, but consider the friction factor for laminar flow as $f = 24/N_{Re}$.

.

### 5.1.5 Frictional Pressure Loss Calculation for Herschel-Bulkley Fluid

To estimate velocity, follow the procedure outlined in Part *a* of Section 5.1.1 (annulus and pipe). [23,26,27]

- Pipe Flow

b. Reynolds number:

$$N_{Re} = \frac{2(3n+1)}{n}\left[\frac{\rho v_p^{(2-n)}\left(\frac{D_p}{2}\right)^n}{\tau_0\left(\frac{D_p}{2v_p}\right)^n + k\left(\frac{3n+1}{nC_c}\right)^n}\right]. \quad \text{.....(5.28)}$$

c. Critical Reynolds numbers value, $N_{Rec}$

$$N_{Rec} = \left[\frac{4(3n+1)}{ny}\right]^{\frac{1}{1-z}}. \quad \text{.....(5.29)}$$

$$y = \frac{\log(n)+3.93}{50}. \quad \text{.....(5.30)}$$

$$z = \frac{1.75-\log(n)}{7}. \quad \text{.....(5.31)}$$

d. Regime flow determination:

Comparison between $N_{Re}$ and $N_{Rec}$

If $N_{Re} < N_{Rec}$ →flow is laminar.

Friction factor included in Eq.5.34

If $N_{Re} > N_{Rec}$ →flow is turbulent.

$$f = y(C_c N_{Re})^{-z}. \quad \text{.....(5.32)}$$

$$C_c = 1-\left(\frac{1}{2n+1}\right)\frac{\tau_0}{\tau_0 + k\left[\frac{(3n+1)q}{n\pi(D_p/2)^3}\right]^n}. \quad \text{.....(5.33)}$$

e. Frictional pressure loss calculation inside the drillstring:

*Laminar:*

$$\left(\frac{dp}{dL}\right)=\frac{4k}{14400D_p}\left\{\left(\frac{\tau_0}{k}\right)+\left[\left(\frac{3n+1}{nC_c}\right)\left(\frac{8q}{\pi D_p{}^3}\right)\right]^n\right\}. \quad \text{.....(5.34)}$$

*Turbulent:*

$$\left(\frac{dp}{dL}\right)=\frac{fq^2\rho}{1421.22D_p{}^5}. \quad \text{.....(5.35)}$$

$$\Delta p_{ds}=\left(\frac{dp}{dL}\right)\Delta L. \quad \text{.....(5.7)}$$

- Annular Flow

b. Reynolds number:

$$N_{\text{Re}}=\frac{4(2n+1)}{n}\left\{\frac{\rho v_a{}^{2-n}\left(\frac{D_2-D_1}{2}\right)^n}{\tau_0\left(\frac{D_2-D_1}{2v_a}\right)^n+k\left[\frac{2(2n+1)}{nC_a^*}\right]^n}\right\}, \quad \text{.....(5.36)}$$

where $v_a$ is annular velocity in ft/sec and $D_1$, $D_2$ are diameters in ft.

c. Critical value $N_{\text{Rec}}$

$$N_{\text{Rec}}=\left[\frac{8(2n+1)}{ny}\right]^{\frac{1}{1-z}}. \quad \text{.....(5.37)}$$

Use Eqs. 5.30 and 5.31 to estimate the values of $z$ and $y$.

d. Regime flow determination:

Comparison between $N_{Re}$ and $NR_{ec}$

If $N_{\text{Re}} < N_{\text{Rec}} \rightarrow$flow is laminar.

Friction factor is included in Eq.5.46

If $N_{\text{Re}} > N_{\text{Rec}} \rightarrow$flow is turbulent.

$$f = y(C_a^* N_{\mathrm{Re}})^{-z}. \quad \text{.....(5.38)}$$

$$C_a^* = 1 - \left(\frac{1}{n+1}\right) \frac{\tau_0}{\tau_0 + k\left\{\dfrac{2q(2n+1)}{n\pi\left[(D_2/2)-(D_1/2)\right]\left[(D_2/2)^2-(D_1/2)^2\right]}\right\}^n}.$$

.....(5.39)

*e*. Frictional pressure loss calculation inside annulus*:*

*Laminar:*

$$\left(\frac{dp}{dL}\right) = \frac{4k}{14400(D_2 - D_1)}\left\{\left(\frac{\tau_0}{k}\right) + \left[\left(\frac{16(2n+1)}{n \times C_a^*(D_2 - D_1)}\right)\left(\frac{q}{\pi(D_2^2 - D_1^2)}\right)\right]^n\right\}.$$

.....(5.40)

*Turbulent:*

$$\left(\frac{dp}{dL}\right) = \frac{fq^2\rho}{1421.22(D_2 - D_1)(D_2^2 - D_1^2)^2}. \quad \text{.....(5.41)}$$

$$\Delta p_a = \left(\frac{dp}{dL}\right)\Delta L. \quad \text{.....(5.7)}$$

### 5.1.6 Frictional Pressure Loss Calculation for Unified Fluid

- Pipe Flow[2,24]

a. Velocity:

$$v_p = \frac{24.5q}{D_p^{\,2}}. \quad \text{.....(5.42)}$$

b. Reynolds number:

$$G = \frac{3n+1}{4n}, \quad \text{.....(5.43)}$$

$$\gamma_w = \frac{1.6Gv_p}{D_p}. \quad \text{.....(5.44)}$$

$$\tau_w = \left(\frac{4}{3}\right)^n \tau_0 + k\gamma_w^n. \quad \text{.....(5.45)}$$

$$N_{\text{Re}} = \frac{\rho v_p^{\,2}}{19.36\tau_w}, \quad \text{.....(5.46)}$$

where G is a unified model parameter, dimensionless.

c. Friction factor determination for any flow regime:

$$f_{\text{laminar}} = 16 / N_{\text{Re}}. \quad \text{.....(5.47)}$$

$$f_{\text{transient}} = \frac{16 N_{\text{Re}}}{(3470 - 1370n)^2}. \quad \text{.....(5.48)}$$

$$a = \frac{\log n + 3.93}{50}. \quad \text{.....(5.19)}$$

$$b = \frac{1.75 - \log n}{7}. \quad \text{.....(5.20)}$$

$$f_{\text{turbulent}} = \frac{a}{N_{\text{Re}}^{\,b}}. \quad \text{.....(5.49)}$$

$$f_{\text{partial}} = \left(f_{\text{transient}}^{-8} + f_{\text{turbulent}}^{-8}\right)^{-1/8}. \quad \text{.....(5.50)}$$

$$f = \left(f_{\text{partial}}^{\,12} + f_{\text{laminar}}^{\,12}\right)^{1/12}. \quad \text{.....(5.51)}$$

d. Frictional pressure loss calculation inside drillstring*:*

$$\left(\frac{dp}{dL}\right) = \frac{1.076 f v_p^{\,2} \rho}{10^5 D_p}. \quad \text{.....(5.52)}$$

$$\Delta p_{ds} = \left(\frac{dp}{dL}\right)\Delta L. \quad \text{.....(5.7)}$$

- Annular Flow

*a.* Velocity*:*

$$v_a = \frac{24.5q}{D_2^{\,2} - D_1^{\,2}}. \quad \text{.....(5.53)}$$

b. Number of Reynolds:

$$G = \left(\frac{2n+1}{3n}\right) \times 1.5 . \quad \text{.....(5.54)}$$

$$\gamma_w = \frac{1.6 G v_a}{D_2 - D_1} . \quad \text{..... (5.55)}$$

$$\tau_w = \left(\frac{3}{2}\right)^n \tau_0 + k\gamma_w^n . \quad \text{.....(5.56)}$$

$$N_{\text{Re}} = \frac{\rho v_a{}^2}{19.36\tau_w} . \quad \text{.....(5.57)}$$

c. Friction factor determination for any flow regime:

$$f_{\text{laminar}} = 16 / N_{\text{Re}} . \quad \text{.....(5.47)}$$

$$f_{\text{transient}} = \frac{16 N_{\text{Re}}}{(3470 - 1370n)^2} . \quad \text{.....(5.48)}$$

$$a = \frac{\log n + 3.93}{50} . \quad \text{.....(5.19)}$$

$$b = \frac{1.75 - \log n}{7} . \quad \text{.....(5.20)}$$

$$f_{\text{turbulen}t} = \frac{a}{N_{\text{Re}}{}^b} . \quad \text{.....(5.49)}$$

$$f_{\text{partial}} = \left(f_{\text{transient}}^{-8} + f_{\text{turbulent}}^{-8}\right)^{-1/8} . \quad \text{.....(5.50)}$$

$$f = \left(f_{\text{partial}}^{12} + f_{\text{laminar}}^{12}\right)^{1/12} . \quad \text{.....(5.51)}$$

d. Frictional pressure loss calculation inside annulus:

$$\left(\frac{dp}{dL}\right) = \frac{1.076 f v_a{}^2 \rho}{10^5 (D_2 - D_1)} . \quad \text{.....(5.58)}$$

$$\Delta p_a = \left(\frac{dp}{dL}\right) \Delta L . \quad \text{.....(5.7)}$$

### 5.1.7 Frictional Pressure Loss Calculation for Robertson and Stiff Fluid

To estimate velocity follow the procedure outline in Part *a* of Section 5.1.6 (annulus and pipe).[26-29]

- Pipe Flow

b. Reynolds number:

$$N_{\mathrm{Re}} = \frac{89100 v_p^{\,2-B} \rho}{A} \left( \frac{0.0416 D_p}{3 + \frac{1}{B}} \right)^B . \quad \text{.....(5.59)}$$

c. For laminar flow,[26] critical value $N_{\mathrm{Re}c} = 3470\text{-}1370B$.

For turbulent flow,[26] critical value $N_{\mathrm{Re}c} = 4270\text{-}1370B$.

d. Regime flow determination:

Comparison between $N_{\mathrm{Re}}$ and $N_{\mathrm{Re}c}$

If $N_{\mathrm{Re}} < N_{\mathrm{Re}c} \rightarrow$flow is laminar.

Friction factor is included in Eq. 5.60.

If $N_{\mathrm{Re}} > N_{\mathrm{Re}c} \rightarrow$flow is turbulent.[26]

$$a = \frac{\log(B) + 3.93}{50} . \quad \text{.....(5.60)}$$

$$b = \frac{1.75 - \log(B)}{7} . \quad \text{.....(5.61)}$$

$$f_{\mathrm{turbulent}} = \frac{a}{N_{\mathrm{Re}}{}^{b}} . \quad \text{.....(5.49)}$$

e. Frictional pressure loss calculation inside drillstring:

*Laminar:*

$$\left( \frac{dp}{dL} \right) = 8.33E - 4 \times 2^{2+B} \times A \left\{ \left( \frac{1+3B}{B} \right) \left[ \frac{0.2 v_p + \frac{C}{6} D_p}{D_p{}^{\left( \frac{1+B}{B} \right)}} \right] \right\}^B . \quad \text{.....(5.62)}$$

*Turbulent:*

To estimate frictional pressure loss for turbulent flow, follow the procedure outlined in Part *e* of Section 5.1.1 (annulus and pipe).

$$\Delta p_{ds} = \left(\frac{dp}{dL}\right)\Delta L \,. \quad \text{.....(5.7)}$$

- Annular Flow

b. Reynolds number:

$$N_{\text{Re}} = \frac{109000 v_a^{\,2-B}\rho}{A}\left(\frac{0.0208(D_2 - D_1)}{2+\frac{1}{B}}\right)^B \,. \quad \text{.....(5.63)}$$

To estimate flow regime, follow the procedure outlined in Parts *c* and *d* of Section 5.1.7 (pipe). For laminar flow in the annulus, use Eq. 5.62.

e. Frictional pressure loss calculation inside annulus:

*Laminar:*

$$\left(\frac{dp}{dL}\right) = 8.33E-4\times 4^{1+B}\times A\left\{\left(\frac{1+2B}{B}\right)\left[\frac{0.2v_a + \frac{C}{8}(D_2 - D_1)}{(D_2 - D_1)^{\left(\frac{1+B}{B}\right)}}\right]\right\}^B \,. \quad (5.64)$$

Note: To consider yield stress with this model, use the following equations and estimate the frictional pressure loss for laminar flow by iteration:

- Pipe Flow

$$\lambda = \frac{2(AC)^B}{\left(\frac{dp}{dL}\right)} \,. \quad \text{.....(5.65)}$$

General equation to estimate friction pressure loss:

$$q = \pi\left\{\left[\frac{1}{2A}\left(\frac{dp}{dL}\right)\right]^{1/B}\left(\frac{B}{3B+1}\right)\left[\left(\frac{D}{2}\right)^{\frac{3B+1}{B}} - \lambda^{\frac{3B+1}{B}}\right] - \frac{C}{3}\left[\left(\frac{D}{2}\right)^{3} - \lambda^{3}\right]\right\}. \quad (5.66)$$

- Annular Flow

$$\lambda = \frac{(AC)^{B}}{\left(\frac{dp}{dL}\right)}. \quad \text{.....(5.67)}$$

$$q = 2D\left\{\left(\frac{1}{2A} * \left(\frac{dp}{dL}\right)\right)^{1/B}\left(\frac{B}{2B+1}\right)\left[\left(\frac{D}{2}\right)^{\frac{3B+1}{B}} - \lambda^{\frac{2B+1}{B}}\right] - \frac{C}{2}\left[\left(\frac{D}{2}\right)^{3} - \lambda^{3}\right]\right\}.$$

.....(5.68)

For the annulus, $D = D_2$-$D_1$ and $D = D_p$ for pipe.

### 5.1.8 Frictional Pressure Loss Calculation for Casson Fluid

To calculate velocity and Reynolds numbers follow the procedure outlined in Parts *a* and *b* of Section 5.1.1 (annulus and pipe).[17,19] Note: Use the Casson viscosity estimate for this model with Eqs. 5.13 and 5.14.

- Pipe Flow

c. Critical Reynolds number value, $N_{Rec}$ from **Fig.5.3.**

$$C_a = \frac{D_p^2 \tau_c \rho}{32.174 \mu_c^2}. \quad \text{.....(5.69)}$$

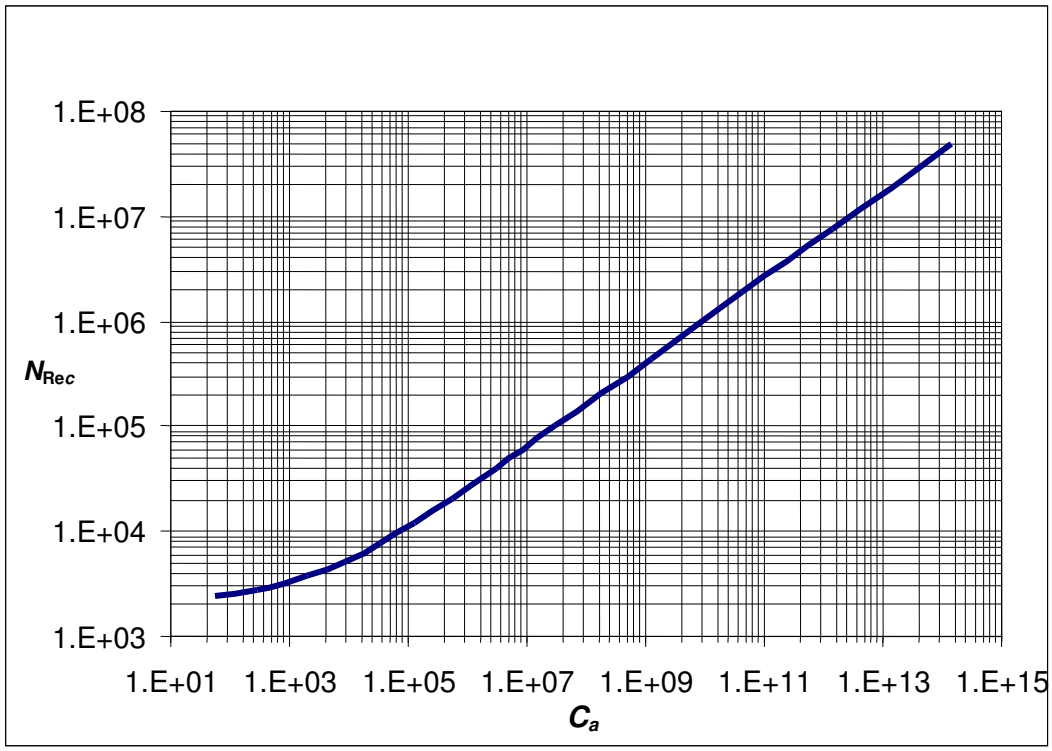



Fig. 5.3— Critical Reynolds numbers for Casson fluids (data from Hanks[30]).

d. Regime flow determination:

Comparison between $N_{Re}$ and $N_{Rec}$

If $N_{Re} < N_{Rec}$ →flow is laminar.

Friction factor is included in Eq. 5.68.

If $N_{Re} > N_{Rec}$ →flow is turbulent.

$$f = \frac{0.0791}{N_{Re}^{0.25}}. \quad \text{.....(5.5)}$$

e. Frictional pressure loss calculation inside drillstring:

*Laminar:*

$$q = \frac{\pi D_p^{\,3}}{8(\mu_c^{\,1/2})^2}\left[\frac{D_p\left(\frac{dp}{dL}\right)}{16} - \frac{4}{7}\sqrt{\tau_c}\sqrt{\frac{\left(\frac{dp}{dL}\right)D_p}{4}} - \frac{64\tau_c^{\,4}}{84D_p^{\,3}\left(\frac{dP}{dL}\right)^3} + \frac{\tau_c}{3}\right] . \ .$$

.....(5.70)

Where

dp/dL= lb/ft$^2$/ft

*Turbulent:*

To estimate frictional pressure loss for turbulent flow, follow the procedure outlined in Part *e* of Section 5.1.1 (annulus and pipe).

$$\Delta p_{ds} = \left(\frac{dp}{dL}\right)\Delta L . \quad \text{.....(5.7)}$$

- Annular Flow

*c.* Critical Reynolds numbers value, $N_{Rec}$ from ***Fig. 5.3***

$$C_a = \frac{(D_2 - D_1)^2 \tau_c \rho}{32.174\mu_c^2}, \quad \text{.....(5.71)}$$

where $D_2$-$D_1$ is the equivalent diameter.

d. Regime flow determination:

Comparison between $N_{Re}$ and $N_{Rec}$

If $N_{Re} < N_{Rec}$ → flow is laminar.

Friction factor is included in Eq. 5.70

If $N_{Re} > N_{Rec}$ → flow is turbulent.

$$f = \frac{0.0791}{N_{Re}^{\,0.25}} . \quad \text{.....(5.5)}$$

e. Frictional pressure loss calculation inside annulus:

*Laminar:*

$$q = \frac{\pi(D_2 - D_1)^3}{8(\mu_c^{1/2})^2} \times$$

$$\left[ \frac{(D_2 - D_1)\left(\frac{dp}{dL}\right)}{16} - \frac{4}{7}\sqrt{\tau_c}\sqrt{\frac{\left(\frac{dp}{dL}\right)(D_2 - D_1)}{4}} - \frac{64\tau_c^{\,4}}{84(D_2 - D_1)^3\left(\frac{dp}{dL}\right)^3} + \frac{\tau_c}{3} \right].$$

.....(5.72)

Where

dp/dL= lb/ft$^2$/ft

$$\Delta p_a = \left(\frac{dp}{dL}\right)\Delta L. \quad \text{.....(5.23)}$$

Note that Eqs.5.70 and 5.72 need Solve from Excel in order to evaluate pressure drop gradient.

# CHAPTER VI
# TOOL JOINT

The tool joint is a necessary part to extend the drillpipe. These components are fabricated separately from the pipe body and welded onto the pipe at a manufacturing facility.

The tool joints provide high-strength, high-pressure threaded connections that are sufficiently robust to survive the rigors of drilling and numerous cycles of tightening and loosening at threads. Tool joints are usually made of steel that has been heat treated to a higher strength than the steel of the tube body.[31]

## 6.1 Weld-On Tool Joint

The flash-welded tool joint was introduced to the industry in 1938 and is now the only tool joint carried in API specifications, **Fig. 6.1**.

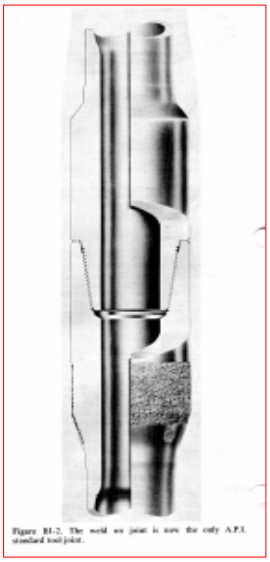


Fig. 6.1— API standards tool joint (from IADC manual[31]).

### 6.1.1 Upset and Designs

Upsets are necessary on drillpipe to which weld-on type tool joints are applied. These allow adequate safety factors in the weld area for mechanical strength and metallurgical considerations. **Fig. 6.2** shows an upset diagram. API upset for various sizes and weights of drillpipe are shown in **Fig. 6.3** and **Tables 6.1** and **6.2.**

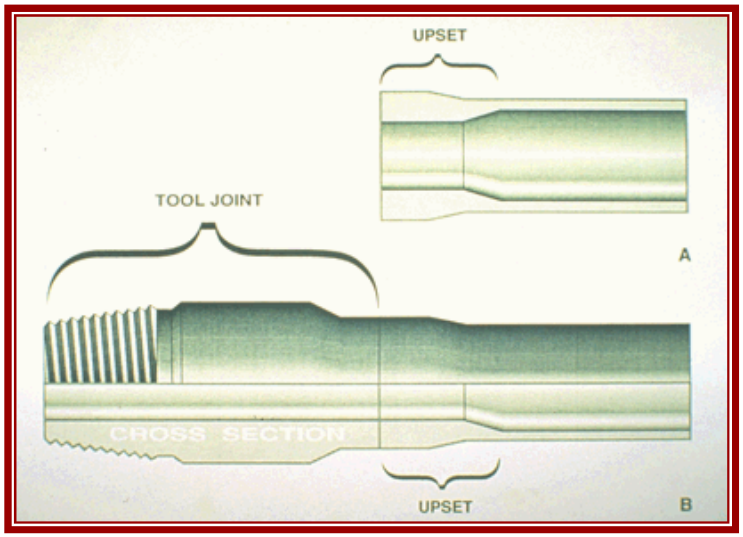



Fig. 6.2— Diagram of tool joint.[32]

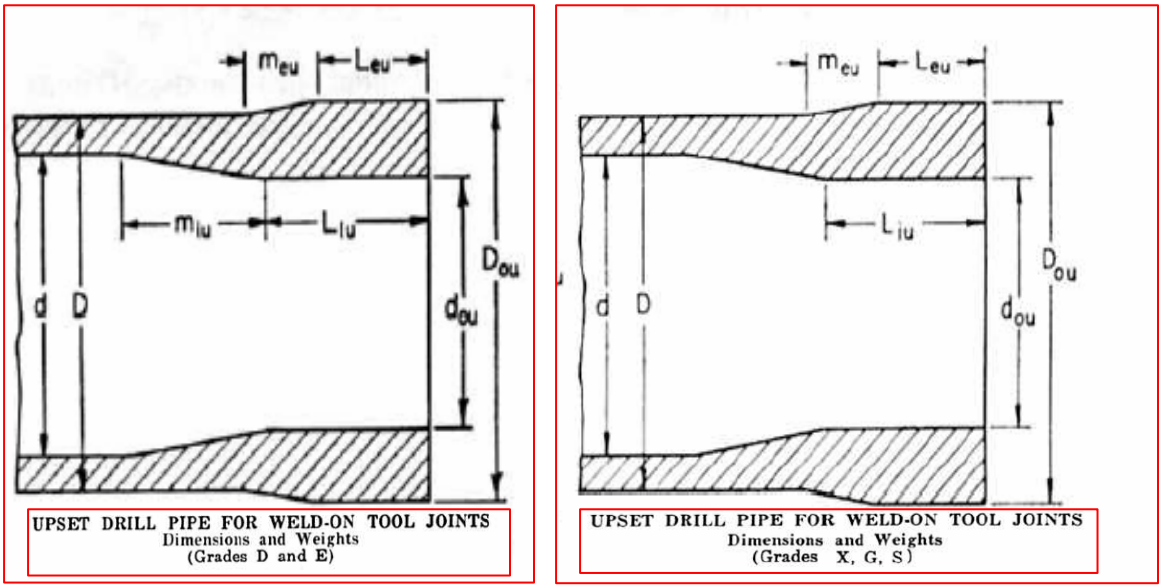



Fig. 6.3— Internal/external upset (from IADC manual[31]).

**Table 6.1— Upset Drillpipe for Weld-On Tool Joints, Grades D and E (from IADC manual[31]).**

| Pipe Size: Outside Dia., in. $D$ | Nominal Wt.:[1] lb/ft | Wall Thickness, in. $t$ | Inside Diameter, in. $d$ | Calculated Weight: Plain End lb/ft $w_{pe}$ | Calculated Weight: Upset[4] lb $e_w$ | [5]Upset Dimensions, in.: Outside Diameter,[2] +⅛, −1/32 $D_{ou}$ | Inside Diameter at End of Pipe,[3] ±1/16 $d_{ou}$ | Length of Internal Upset +1½ − ½ $L_{iu}$ | Length of Internal Taper, min. $m_{iu}$ | Length of External Upset, min. $L_{eu}$ | Length of External Taper, min. | Length of External Taper, max. $m_{eu}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 4½ | 20.00 | 0.430 | 3.640 | 18.69 | 8.60 | 4.781 | 3 | 2¼ | 2 | 1½ | 1 | 1½ |
| 5 | 19.50 | 0.362 | 4.276 | 17.93 | 8.60 | 5.188 | 3 11/16 | 2¼ | 2 | 1½ | 1 | 1½ |
| 5 | 25.60 | 0.500 | 4.000 | 24.03 | 7.80 | 5.188 | 3 7/16 | 2¼ | 2 | 1½ | 1 | 1½ |
| 5½ | 21.90 | 0.361 | 4.778 | 19.81 | 10.60 | 5.563 | 4 | 2¼ | 2 | 1½ | 1 | 1½ |
| 5½ | 24.70 | 0.415 | 4.670 | 22.54 | 9.00 | 5.563 | 4 | 2¼ | 2 | 1½ | 1 | 1½ |

**Table 6.2— Upset Drillpipe for Weld-On Tool Joints, Grades X, G and S (from IADC manual[31]).**

| Pipe Size: Outside Dia., in. $D$ | Nominal Wt.,[1] lb/ft | Wall Thickness, in. $t$ | Inside Diameter, in. $d$ | Calculated Weight: Plain End lb/ft $w_{pe}$ | Calculated Weight: Upset[4] lb $e_w$ | [5]Upset Dimensions, in.: Outside Diameter,[2] $+\frac{1}{8}$, $-\frac{1}{32}$ $D_{ou}$ | [5]Upset Dimensions, in.: Inside Diameter at End of Pipe,[3] $\pm\frac{1}{16}$ $d_{ou}$ | [5]Upset Dimensions, in.: Length of Internal Upset $+1\frac{1}{2}$ $-\frac{1}{2}$ $L_{iu}$ | [5]Upset Dimensions, in.: Length of External Upset, min. $L_{eu}$ | [5]Upset Dimensions, in.: Length End of Pipe to Taper Fadeout, Ext. Upset max. $L_{eu}+m_{eu}$ |
|---|---|---|---|---|---|---|---|---|---|---|
| 3½ | 15.50 | 0.449 | 2.602 | 14.63 | 11.00 | 3.781 | 1 15/16 | 4¼ | 3 | 5½ |
| 4½ | 20.00 | 0.430 | 3.640 | 18.69 | 17.60 | 4.781 | 2 13/16 | 4¼ | 3 | 5½ |
| 5 | 19.50 | 0.362 | 4.276 | 17.93 | 16.80 | 5.188 | 3 9/16 | 4¼ | 3 | 5½ |
| 5 | 25.60 | 0.500 | 4.000 | 24.03 | 15.40 | 5.188 | 3 5/16 | 4¼ | 3 | 5½ |
| 5½ | 21.90 | 0.361 | 4.778 | 19.81 | 21.00 | 5.563 | 3 13/16 | 4¼ | 3 | 5½ |
| 5½ | 24.70 | 0.415 | 4.670 | 22.54 | 18.40 | 5.563 | 3 7/16 | 4¼ | 3 | 5½ |

Deepwater drilling necessitates high strength drillpipe which often has small throated (internal upset) tool joints.

**Figs. 6.4 and 6.5** show the pin and box areas, which are the largest factor and are subject to the widest variation. The tool joint outside diameter (OD) and inside diameter (ID) largely determine the strength of the joint in torsion. The OD affects the box area the ID affects the pin area. Choice of OD and ID determines the areas of the pin and box and establishes the theorical torsional strength.

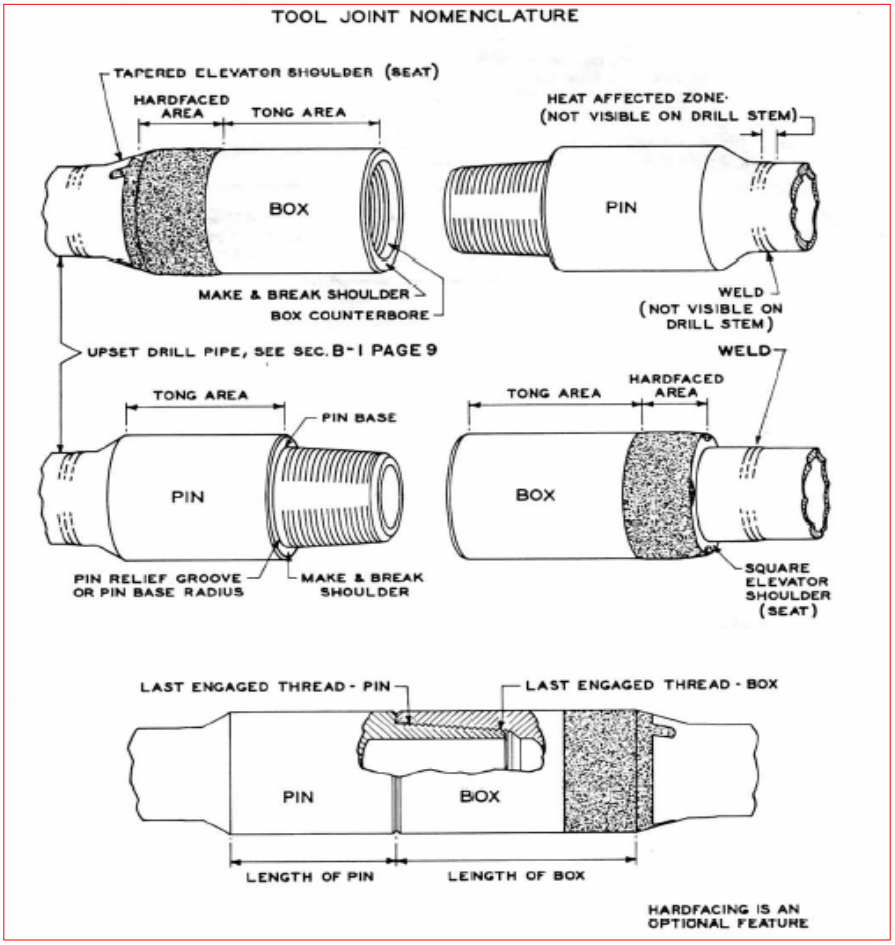



Fig. 6.4— Tool joint nomenclature (from IADC manual[31]).

### 6.1.2 Cleaning and Inspection

Pin and box threads and shoulders should be thoroughly cleaned in preparation to adding them to the string. Cleaning pays off in three ways. First, it removes foreign material and permits proper make-up, thereby reducing danger of galling and wobbles. Second, it permits better inspection. Third, it increases life of connections by elimination of abrasive materials. Connections should be carefully dried after cleaning so that the thread compound will properly adhere to the surface.

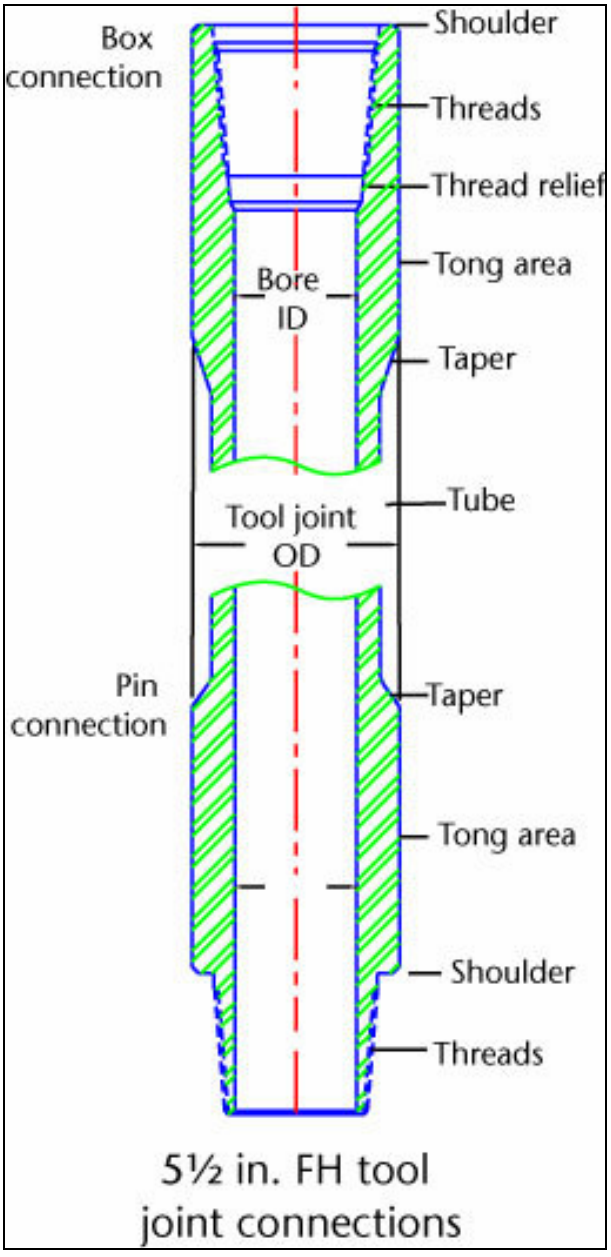



Fig. 6.5 —Schematic of the box and pin ends of a joint of pipe. ID = internal diameter, OD = outside diameter, FH = full hole.[32]

# CHAPTER VII
# STUDY APPROACHES TO ESTIMATE PRESSURE LOSSES BY CORRECTING FOR TOOL JOINT LOSSES

The prediction of friction pressure losses is important in many field operations, including drilling, completation, fracturing, acidizing, workover and production. Also, deep drilling necessitates high-strength drillpipe which often has small-throated (internal upset) tool joints. These internal limitations cause flow losses which can be considerable. The pressure loss caused by entry into the tool joint is small compared with the exit losses.

On the other hand, external upset of tool joint can cause the same problem in the annulus between tool joint and casing. This space is narrower than the space between drillpipe and casing. As a result, the expansion and contraction of the annulus during fluid flow causes additional pressure loss.

This research proposes five approaches to correct pump pressure loss by tool joints, and Appendix C give an example of how these approaches work:

1. Enlargement and contraction (E&C).

2. Equivalent diameter (ED).

3. Two different IDs (2IDs).

4. Enlargement and contraction plus equivalent diameter (E&C+ED).

5. Enlargement and contraction plus two different IDs (E&C+2IDs).

## 7.1 Enlargement and Contraction

When fluid is flowing steadily in a long, straight pipe of uniform diameter, the flow pattern, as indicated by the velocity distribution across the pipe diameter, will assume a certain characteristic form. Any impediment in the pipe which changes the direction of the whole stream, or even part of it, will alter the characteristic flow pattern and create turbulence, causing an energy loss greater than that normally accompanying flow in straight pipe. This disturbance in the flow pattern produces an additional pressure drop.[33]

### 7.1.1 Gradual Enlargement for Pipe

The losses due to gradual enlargement of pipes were investigated by Gibson,[34] and **Fig. 7.1** shows the geometry's change. Also, the resistance to flow may be expressed by the coefficient $K_e$. See Eq. 7.1.

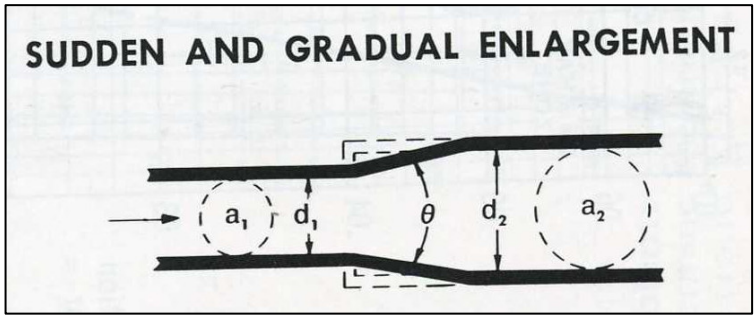



Fig. 7.1 —Schematic change of area, $a_1$ and $a_2$, in a pipe with a tool joint (from tool joint to pipe).[33]

***45°<θ≤180°***

$$K_e = (1-\beta^2)^2, \quad \text{.....(7.1)}$$

***θ≤45°***

$$K_e = 2.6\sin\left(\frac{\theta}{2}\right)(1-\beta^2)^2, \quad \text{.....(7.2)}$$

where *β* is the ratio of diameters of small to large pipes, dimensionless.

The mechanical energy loss, $F_e$, between two different successive diameters can be expressed by comparing Bernoulli equation at two points. See Eq.7.2

$$F_e = K_e\left(\frac{v^2}{2gc}\right). \quad \text{.....(7.3)}$$

The pressure loss then is calculated by multiplying the fluid density by mechanical energy loss for gradual enlargements.

$$\Delta p_e = 0.052 F_e \rho. \quad \text{.....(7.4)}$$

**7.1.2 Gradual Contraction for Pipe**

The same procedure is followed to obtain the pressure loss for gradual contraction. See **Fig. 7.2**.

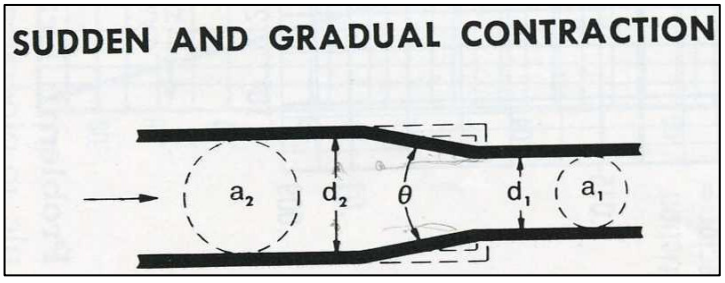



Fig. 7.2 —Schematic change of area, $a_1$ and $a_2$, in a pipe with a tool joint (enter from pipe to tool joint).[33]

***45°<θ≤180°***

$$K_c = 0.5\sqrt{\sin\left(\frac{\theta}{2}\right)}(1-\beta^2) \quad \text{.....(7.5)}$$

***θ≤45°***

$$K_c = 0.8\sin\left(\frac{\theta}{2}\right)(1-\beta^2) \quad \text{.....(7.6)}$$

Then;

$$F_c = K_c\left(\frac{v^2}{2gc}\right) \quad \text{.....(7.7)}$$

$$\Delta p_c = 0.052 F_c \rho \quad \text{.....(7.8)}$$

Note the convergence or divergence angle can be estimated using tables and figures in Chapter VI. Also, see **Fig. 7.3** and **Tables 7.1** and **7.2.**

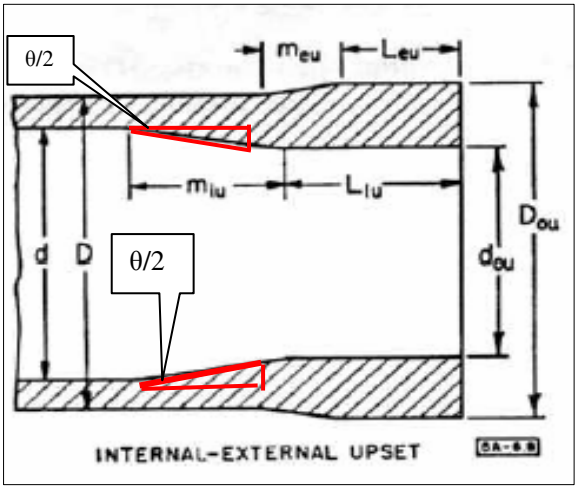



Fig. 7.3 —The angle, *θ*, is an important element in the enlargement and contraction equations.[33]

**Table 7.1— Angles for Internal Upset (Drillpipe) for Weld-On Tool Joints**

| Pipe | | | | tool joint | | | angle calculation | |
|---|---|---|---|---|---|---|---|---|
| OD,in | wt(lbf/ft) | Grade | d,in | miu,in | dou,in | d-dou | θ/2 | θ |
| 4.5 | 20 | D,E | 3.6400 | 2 | 3.0000 | 0.6400 | 17.7500 | 35.5000 |
| 5 | 19.5 | D,E | 4.2760 | 2 | 3.6875 | 0.5885 | 16.4000 | 32.8000 |
| 5 | 25.6 | D,E | 4.0000 | 2 | 3.3750 | 0.6250 | 17.3500 | 34.7000 |
| 5.5 | 21.9 | D,E | 4.7780 | 2 | 4.0000 | 0.7780 | 21.2500 | 42.5000 |
| 5.5 | 24.7 | D,E | 4.6700 | 2 | 4.0000 | 0.6700 | 18.5200 | 37.0400 |
| 3.5 | 15.5 | X,G,S | 2.6020 | 2 | 1.9375 | 0.6645 | 18.3800 | 36.7600 |
| 4.5 | 20 | X,G,S | 3.6400 | 2 | 2.8125 | 0.8275 | 22.4800 | 44.9600 |
| 5 | 19.5 | X,G,S | 4.2760 | 2 | 3.5625 | 0.7135 | 19.6300 | 39.2600 |
| 5 | 25.6 | X,G,S | 4.0000 | 2 | 3.3125 | 0.6875 | 18.9600 | 37.9200 |
| 5.5 | 21.9 | X,G,S | 4.7780 | 2 | 3.8125 | 0.9655 | 25.7700 | 51.5400 |
| 5.5 | 24.7 | X,G,S | 4.6700 | 2 | 3.8125 | 0.8575 | 23.2100 | 46.4200 |

**Table 7.2— Angles for External Upset (Annulus) for Weld-On Tool Joints**

| Pipe | | | | tool joint | | | angle calculation | |
|---|---|---|---|---|---|---|---|---|
| D,in | wt(lbf/ft) | Grade | d,in | meu,in | Dou,in | Dou-D | θ/2 | θ |
| 4.5 | 20 | D,E | 3.6400 | 1.5 | 4.7810 | 0.2810 | 10.6100 | 21.2200 |
| 5 | 19.5 | D,E | 4.2760 | 1.5 | 5.1880 | 0.1880 | 7.1400 | 14.2800 |
| 5 | 25.6 | D,E | 4.0000 | 1.5 | 5.1880 | 0.1880 | 7.1400 | 14.2800 |
| 5.5 | 21.9 | D,E | 4.7780 | 1.5 | 5.5630 | 0.0630 | 2.4000 | 4.8000 |
| 5.5 | 24.7 | D,E | 4.6700 | 1.5 | 5.5630 | 0.0630 | 2.4000 | 4.8000 |
| 3.5 | 15.5 | X,G,S | 2.6020 | 2.5 | 3.7810 | 0.2810 | 6.4100 | 12.8200 |
| 4.5 | 20 | X,G,S | 3.6400 | 2.5 | 4.7810 | 0.2810 | 6.4100 | 12.8200 |
| 5 | 19.5 | X,G,S | 4.2760 | 2.5 | 5.1880 | 0.1880 | 4.3000 | 8.6000 |
| 5 | 25.6 | X,G,S | 4.0000 | 2.5 | 5.1880 | 0.1880 | 4.3000 | 8.6000 |
| 5.5 | 21.9 | X,G,S | 4.7780 | 2.5 | 5.5630 | 0.0630 | 1.4400 | 2.8800 |
| 5.5 | 24.7 | X,G,S | 4.6700 | 2.5 | 5.5630 | 0.0630 | 1.4400 | 2.8800 |

### 7.1.3 Gradual Enlargement and Contraction for Annulus

The procedure is the same as followed in Sections 7.1.1 and 7.1.2. However, notice that the velocity used to estimate the pressure loss by enlargement and contraction corresponds to the narrow annulus. See **Fig. 7.4**.

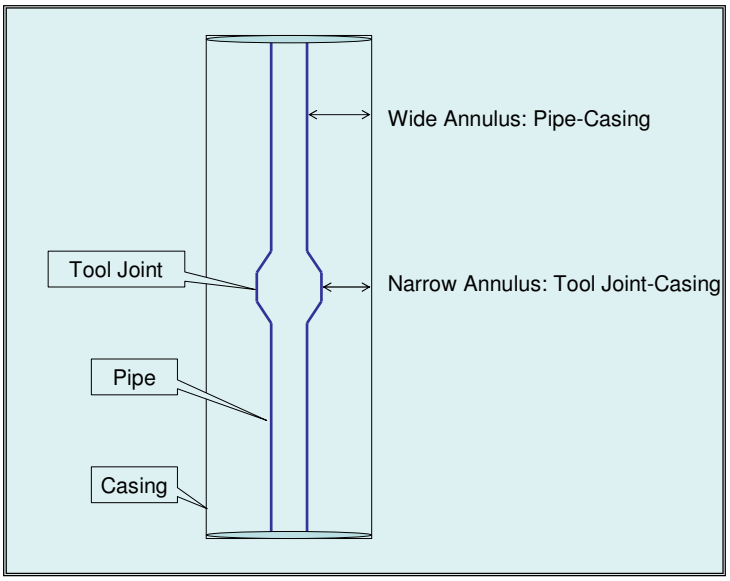



Fig. 7.4 —Schematic change of area in the annulus with presence of tool joint.

### 7.1.4 Estimation of Pump Pressure Considering Enlargement and Contraction Correction

Add to drillstring friction pressure losses calculated (with any correction) the pressure losses caused by enlargement and contraction of each tool joint. Do the same for the annulus friction pressure losses.

$$\Delta p_p=\Delta p_s+[\Delta p_{ds}+(\Delta p_{e,}+\Delta p_c)N_{TJ}]+[\Delta p_a+(\Delta p_e+\Delta p_c)N_{TJ}]+\Delta p_b. \quad (7.9)$$

### 7.2 Equivalent Diameter

Equivalent diameter is a technique that makes an adjustment between two diameters. Consider internal drillpipe and tool joint diameters to estimate pressure drop calculation in the drillstring and external drill pipe and tool joint diameters for the annulus.

Use the following equation to estimate equivalent diameter[§], $D_e$, in drillstring (between inside pipe and tool joint diameters).

$$De_p = \left[ \frac{L_2 d_{TJ}{}^4 D_P^4}{L_1 D_P^4 + (L_1 - L_2) d_{TJ}^4} \right]^{1/4} . \quad \text{.....(7.10)}$$

Use Eq. 7.11 to estimate equivalent diameter in the annulus (between outside pipe and tool joint diameters).

$$De_a = \left[ \frac{L_2 D_{TJ}{}^4 D_1^4}{L_1 D_1^4 + (L_1 - L_2) D_{TJ}^4} \right]^{1/4} . \quad \text{.....(7.11)}$$

Finally, calculate friction pressure losses in the drillstring and annulus as do normally but use equivalent diameter in the calculation of frictional pressure drop.

$$\Delta p_p = \Delta p_s + \Delta p_{ds} + \Delta p_a + \Delta p_b. \quad \text{.....(5.2)}$$

### 7.3 Two Different IDs

This approach proposes to estimate the frictional pressure drop in the annulus and in the drillstring considering the actual pipe/tool joint length and diameter in the calculation.

---

[§]Personal communication, C.Brian. Grant Prideco, USA,TX. Oct. 2005.

- Pipe:

a. Estimation of total drillstring length, $L_{\text{total } dp}$:

$$L_{\text{total } dp} = (L_2 N_{DP} - L_1 N_{TJ}). \quad \text{.....(7.12)}$$

b. Use the ID of the drillstring to estimate the frictional pressure drop, $\Delta p_{ds}$:

$$\Delta p_{ds} = (dp/dL)_{ds} L_{\text{total } dp}. \quad \text{.....(7.13)}$$

c. Estimation of total tool joint length, $L_{\text{total } TJ}$:

$$L_{\text{total } TJ} = (L_1 N_{TJ}). \quad \text{.....(7.14)}$$

d. Use ID of tool joint and respective length to calculate its contribution to the pressure loss to the drillstring, $\Delta p_{TJ}$.

$$\Delta p_{TJ} = (dp/dL)_{TJ} L_{\text{total } TJ}. \quad \text{.....(7.15)}$$

e. Add drillstring and tool joint frictional pressure drop to estimate the total drillstring friction pressure losses.

$$(\Delta p_{\text{total } 2IDs})_{ds} = \Delta p_{ds} + \Delta p_{TJ}. \quad \text{.....(7.16)}$$

- Annulus:

Use the same procedure to estimate the total frictional pressure drop in the annulus, $(\Delta p_{\text{total } 2IDs})_a$, followed in the pipe section, but use the annulus data.

Finally,

$$\Delta p_p = \Delta p_s + (\Delta p_{\text{Total } 2IDs})_{ds} + (\Delta p_{\text{Total } 2IDs})_a + \Delta p_b \text{.....(7.17)}$$

### 7.4 Enlargement and Contraction Plus Equivalent Diameter

The main idea in this approach is to combine the first two methods to correct frictional pressure drop together. To reach this objective, use the following procedure:

a. Estimate the contribution to frictional pressure drop in the drillstring and in the annulus by enlargement and contraction. See Section 7.1.
b. Calculate the frictional pressure drop in the drillstring and in the annulus as shown in Section 7.2.
c. Add the enlargement and contraction contribution to frictional pressure drop in the drillstring and in the annulus already corrected by equivalent diameter.

### 7.5 Enlargement and Contraction Plus Two Different IDs

In this case we evaluate two approaches together one more time. To achieve this goal apply the following steps:

a. Estimate the contribution to frictional pressure drop in the drillstring and in the annulus by enlargement and contraction. See Section 7.1
b. Calculate the frictional pressure drop in the drillstring and in the annulus as shown in Section 7.3.
c. Add the enlargement and contraction contribution to frictional pressure drop in the drillstring and in the annulus already corrected by two different IDs.

# CHAPTER VIII
# DATA USED TO VALIDATE THE NEW APPROACHES

Accurate downhole and surface measurements of a synthetic-based drilling fluid were taken in a Gulf of Mexico well to determine variances between actual and calculated pump pressure.

A special team headed by Marathon Oil Co. successfully instrumented and collected a very large volume of hydraulics data on a well in the Gulf on Mexico at 12,710 ft measured depth. Using multiple sensor packages, accurate measurement of downhole dynamic pressure (hydraulic data) were obtained.[9]

The well selected for the test was in 420 ft of water in Block 89, South Pass, Gulf of Mexico. Testing was conducted after running and cementing a single-weight intermediate string of 11 7/8-in. casing to 12,710 ft. **Fig. 8.1** shows the well profile at the time of the test with the 5-in. drillstring run to 12,439 ft measured depth. Drillstring details also are given in **Fig. 8.1.**

The mud was the same 11.5 lbm/gal polyaphaolefin (PAO)-based synthetic drilling fluid used to drill the long, intermediate casing interval. A single mud pit was isolated to limit surface volume to about 220 bbl and to minimize circulating time for conditioning mud. This also reduces temperature variations while the mud was on the surface. The temperature seems to be constant during the test and it is approximate to 150°F.

In addition to conventional rheological measurements, HPHT properties were taken using a Fann Model 70 viscometer.

**Table 4.1** shows the data obtained from White and Zamora[9] used in the project.

**Table 4.1—Data From Fann 70 (from White and Zamora[9])**

| RPM | Reading |
|---|---|
| 600 | 92 |
| 300 | 58 |
| 200 | 46 |
| 100 | 32 |
| 6 | 10 |
| 3 | 8 |

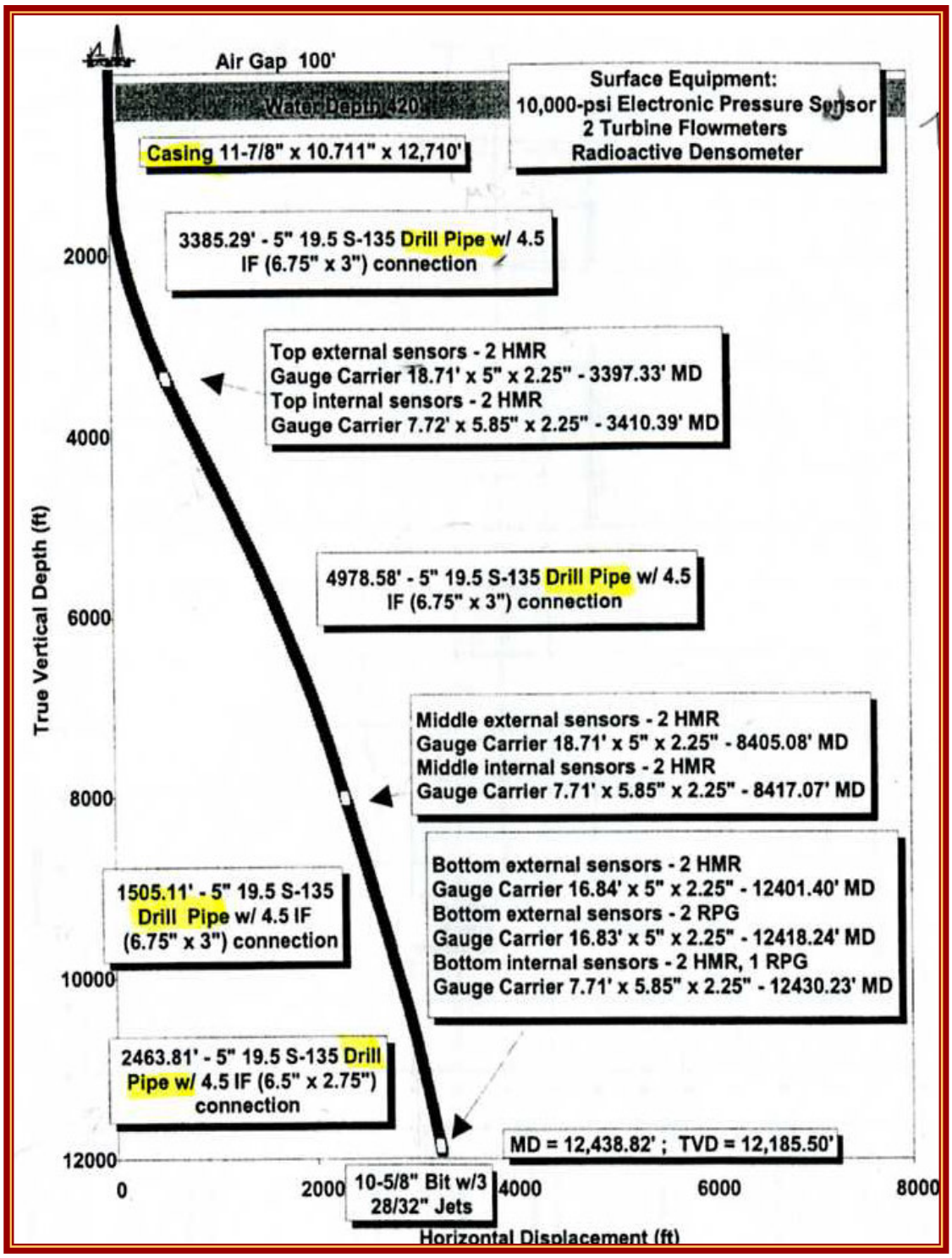



Fig. 8.1— Schematic of well and instrumentation for hydraulics study.[9]

# CHAPTER IX
# RESULTS ANALYSIS

One set of data of non-Newtonian drilling fluid has been used to illustrate the accuracy of this approach for selecting the best rheological model, one with the lowest $E_{AAP}$ value. The physical properties of this fluid are given in **Table 4.1**, and the values for the absolute average percent error are given for each model in **Table 9.1**.

**Table 9.1—$E_{AAP}$ Value for Rheological Models**

| Rheological Model | $E_{AAP}$ |
|---|---|
| API | 1.510 |
| Herschel & Bulkley | 2.898 |
| Robertson & Stiff | 2.914 |
| Unified | 3.952 |
| Casson | 4.667 |
| Power Law | 6.887 |
| Bingham | 24.261 |
| Newtonian | 46.538 |

**Table 9.1** shows that the API RP13D model was the best model to represent the rheological properties for this non-Newtonian fluid. However, it was close followed by Herschel-Bulkley, Casson, Robertson and Stiff, and Unified.

The Newtonian and Bingham models gave high values of $E_{AAP}$ for the fluid, and therefore they are not recommended for use in pressure drop and hydraulics calculations. See appendix D.

Drilling fluid viscosity has a significant impact on circulating pressure losses and solid suspension characteristics of the fluid.
On the other hand, we used five tool joint corrections for this set of data for all rheological models. The data used consisted of rheological and pressure friction loss in the drillstring and annulus, and pump pressure. We compared the calculated drillstring, annulus, and pump pressures, which were corrected, with the drillstring, annulus and pump pressure measured by White and Zamora.[9]

The eight rheological models gave better idea of how the correction by the presence of tool joints can influence the pump pressure for each one.

**Fig. 9.1** shows the data of pump pressure vs. measured and calculated flow rates. Note that the calculated pump pressure is derived without any tool joint corrections.

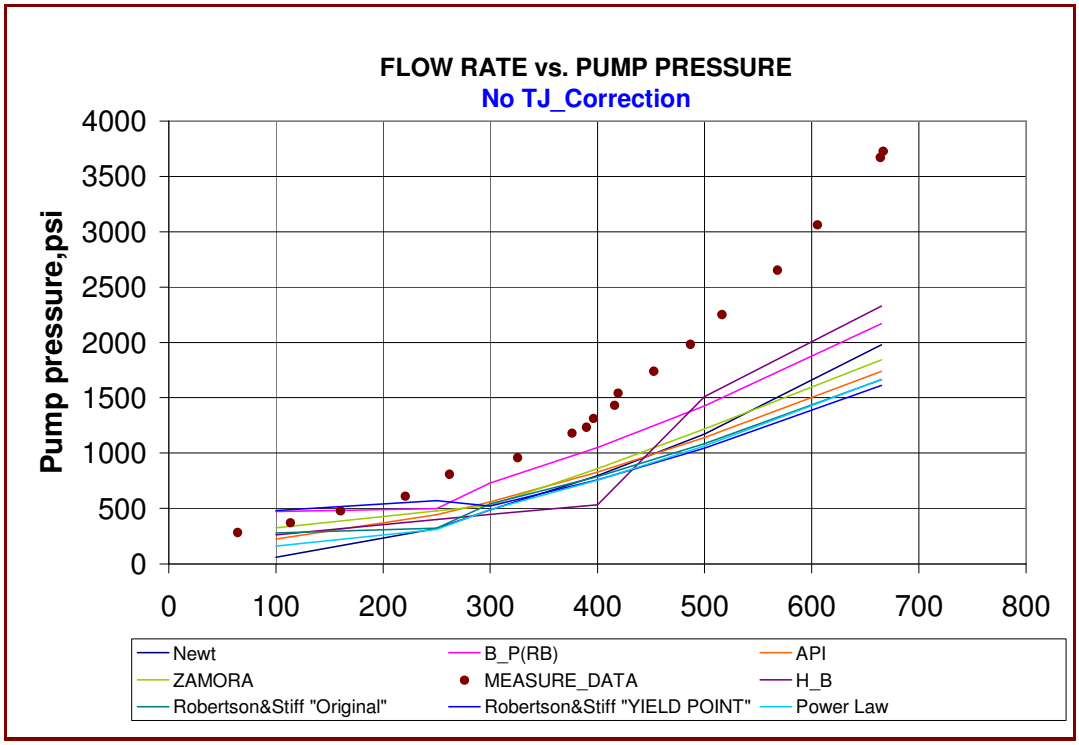



Fig. 9.1— Flow rate vs. pump pressure for eight rheological models at 150°F.

For this case, with the API RP 13D we found a relative error of 42 % between the measured and calculated pump pressure. Also, the best approximation was for the Bingham plastic model with 28%.

Now, let's consider the first approach, correction by enlargement and contraction, in the analysis. **Fig. 9.2** shows how the calculated pump pressure matches the measured data. API RP 13D presents a good fit with 10% relative error. However, the best match can be achieved with the unified model (6%). Note that the relative error consideration of enlargement and contraction significantly reduces the relative error.

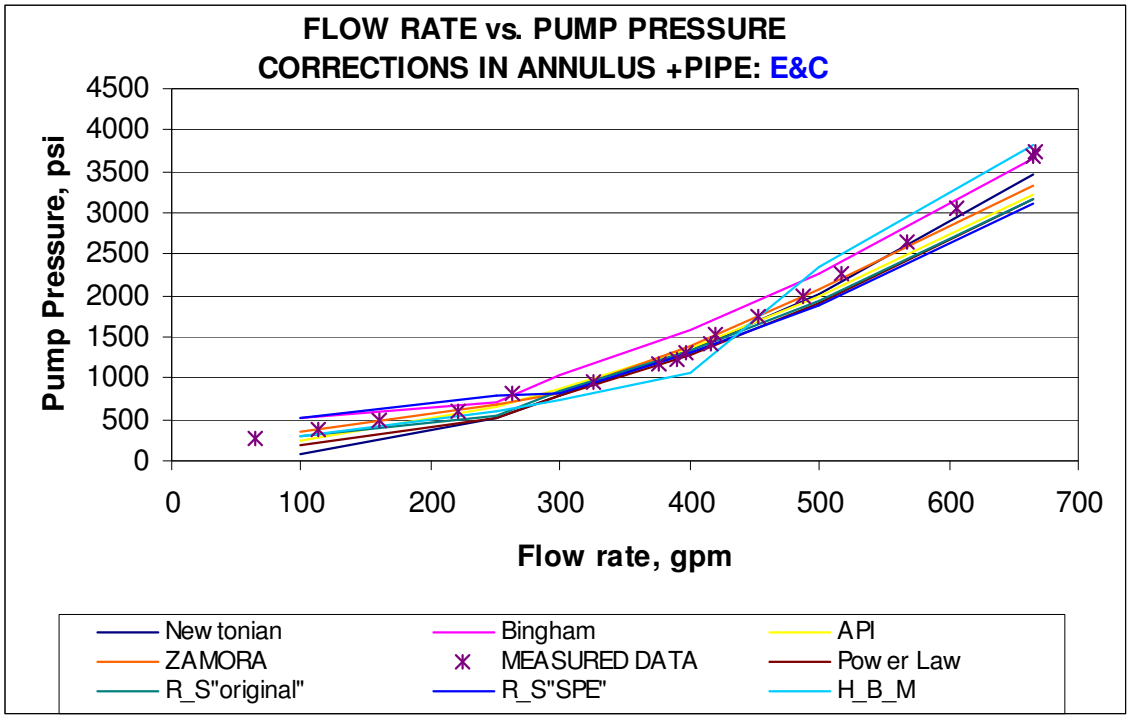



Fig. 9.2— Flow rate vs. pump pressure with E&C correction.

**Fig. 9.3** shows that the match between pump calculated and measured pump pressure is not as good as the first approach when corrections depend on equivalent diameter. However, the improvement over uncorrected approaches is clear. All models except for Robertson and Stiff (original and yield point) present a good match; relative errors for this case are in a range of 54 to 62%.

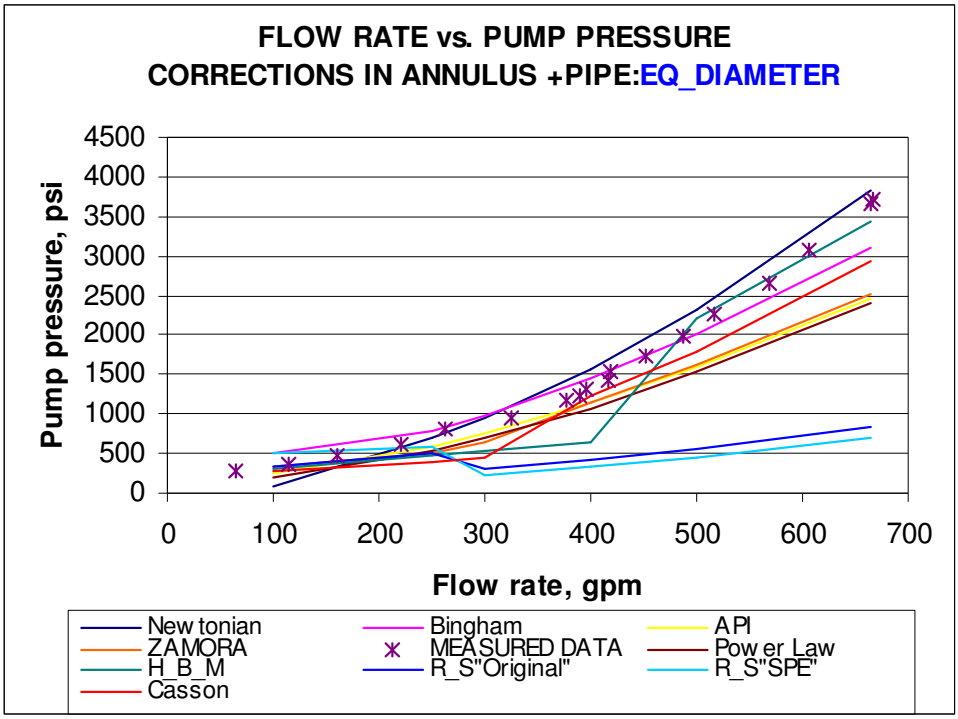



Fig. 9.3— Flow rate vs. pump pressure with ED correction.

**Fig. 9.4** shows that the match between calculated and measured pump pressure is not as good as the first and the second approaches when two different IDs are considered. However, progress is obvious. The Casson model presents the best adjustment.

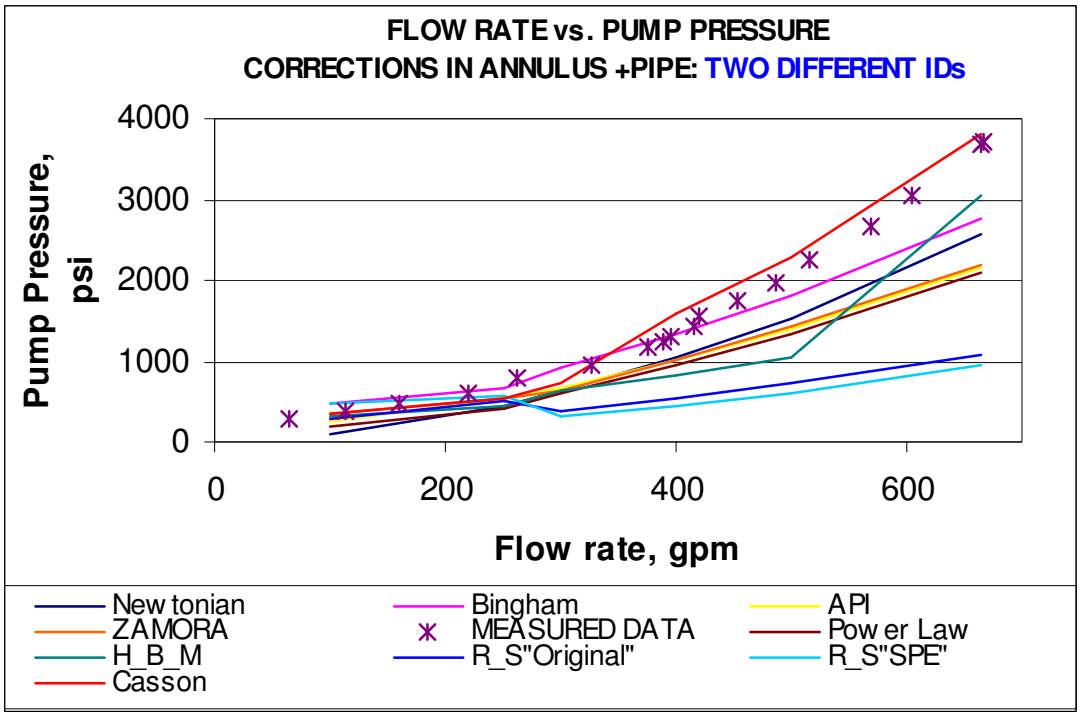



Fig. 9.4— Flow rate vs. pump pressure with correction for 2IDs

Now, let's consider the fourth approach, correction by enlargement and contraction plus equivalent diameter, in the analysis. **Fig. 9.5** shows how the calculated pump pressured matches the measured data. The API model presents a good fit with 15% relative error. However the best match can be achieved with the Unified model (12%).

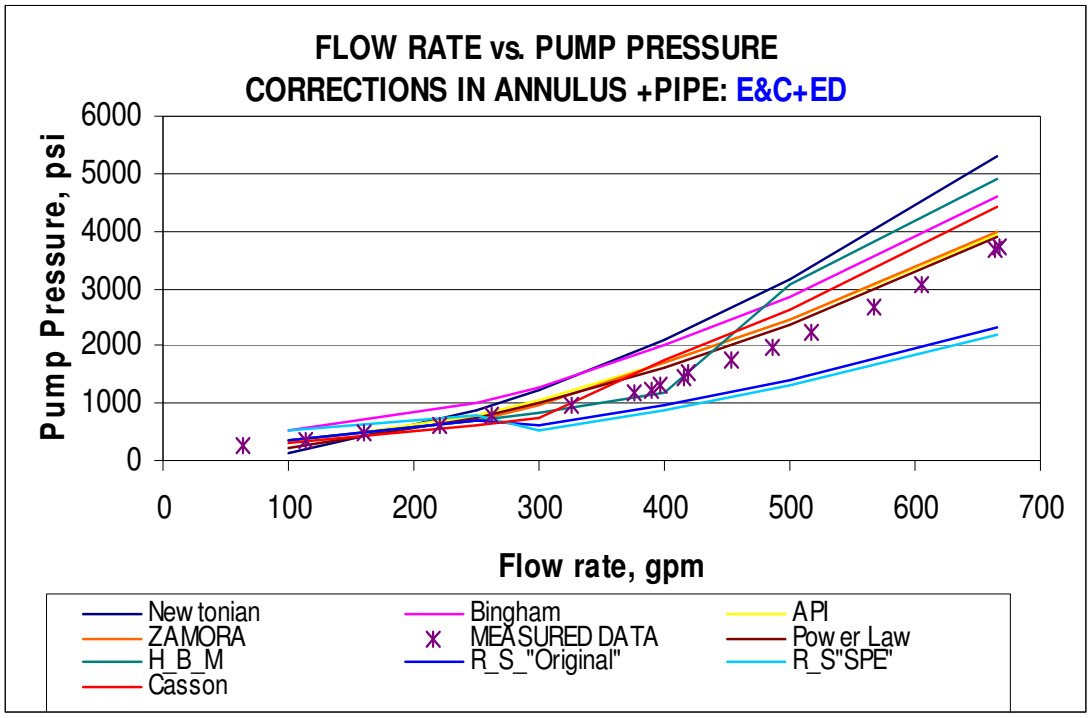



Fig. 9.5— Flow rate vs. pump pressure with correction for E&C+ ED.

Considering enlargement and contraction plus two different IDs in the fifth approach (**Fig. 9.6)**, the match between calculated and measured pump pressure is really good. Also, this approach gives the best adjustment in comparison with the earlier approaches. The best rheological model that matches with the measured data are the API model with 9% relative error, the Unified model with 7% relative error and Herschel-Bulkley model with 8% relative error.

**Table 9.2** shows more details.

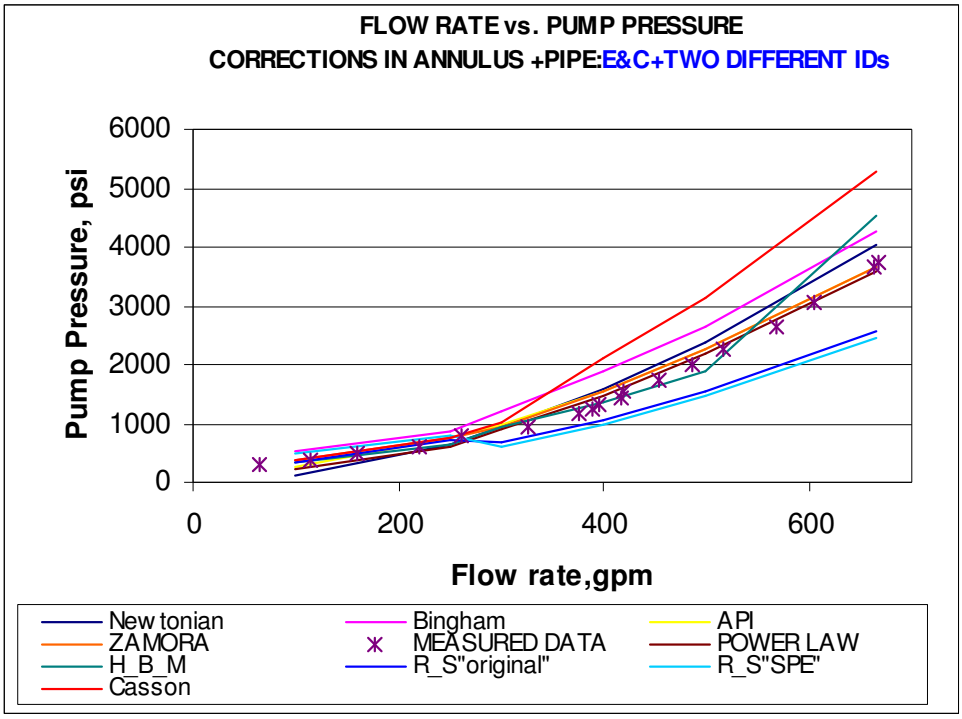



Fig. 9.6— Flow rate vs. pump pressure with correction for E&C+2IDs.

**Table 9.2— Analysis of Five Corrections for Eight Rheological Models**

| RHEOLOGICAL MODELS, T=150 F | N_C | E&C | TWO_ID | EQ_DIAM | E&C+TWO_ID | E&C+EQ_DIAM |
|---|---|---|---|---|---|---|
| NEWTONIAN MODEL | 52 | 20 | 37 | 21 | 21 | 47 |
| BINGHAM PLASTIC MODEL | 28 | 17 | 16 | 23 | 31 | 55 |
| POWER LAW MODEL | 51 | 18 | 39 | 29 | 13 | 15 |
| API MODEL | 42 | 10 | 30 | 22 | 9 | 15 |
| UNIFIED ZAMORA MODEL | 35 | 6 | 25 | 22 | 7 | 12 |
| HERSCHEL & BULKLEY MODEL | 41 | 14 | 32 | 25 | 9 | 18 |
| ROBERTSON &STIFF"original" | 43 | 11 | 50 | 54 | 18 | 23 |
| ROBERTSON &STIFF"yield point" | 42 | 15 | 56 | 62 | 29 | 36 |
| CASSON MODEL | 40 | 8 | 14 | 15 | 29 | 21 |

Let’s analyze what happened in the annulus. Inside the annulus, the velocity is lower if we compare it with the drillstring; as a consequence, the shear rate is reduced and the viscosity is increased.

Most of the cases, the drilling fluid inside the annulus behaves as shear-thinning fluid presenting a yield stress as a viscoplastic fluid does. This is the reason why the rheological models that can be simulated as shear-thinning with yield point have better results.

This is the case the shear-thinning Power Law model gives the best approximation. See Appendix E for more details.

# CHAPTER X

# CONCLUSIONS AND RECOMMENDATIONS

## 10.1 Conclusions

- The rheological models, Bingham, Power-law, API RP 13D, Herschel-Bulkley, Unified, Robertson and Stiff, and Casson have been evaluated for accurate representation of the wide range of shear stress/shear rate data. These models are confirmed to describe sufficiently the rheology of most non-Newtonian fluids.

- Selection of the best rheological model is of great importance in obtaining correct results for pressure drop and hydraulics. A simple and direct approach has been presented for selecting the best rheological model for any non-Newtonian fluid according to the lowest $E_{AAP}$ criteria.

- The Casson model can be applied with high confidence to predict rheological properties and hydraulics calculations in oil-based mud. Also, this model can fit adequately many real yield stress fluids, with simply two parameters.

- The API model provides the best general prediction of rheological behavior for the mud samples studied. It was followed by Herschel-Bulkley, Robertson and Stiff, and Unified models. Correct pressure drop calculations can be achieved by using the $E_{AAP}$ approach, since these calculations depend mainly on the selected model.

- A tool joint can increase pressure losses in the annulus and in the drillstring due to geometry effects of contraction and expansion. Current

API recommended drilling hydraulics calculation techniques (from RP 13D) do not include tool-joint parameters; the API calculations are not accurate. To dramatically reduce errors, practical methods have been developed to correct the pump pressure friction losses including the by tool joint effect.

- The proposed methods for predicting pressure losses by correcting for tool joint effects using Herschel-Bulkley, Unified, Casson and Robertson and Stiff models work well. The results were more accurate than those obtained with standards method using Bingham plastic and Power-law models.

- For mud samples studied, the E&C+2IDs and Unified model (after correction), followed by Herschel-Bulkley and API, and give the best approximation to measured pump pressure.

**10.2 Recommendations**

Considering other data sets is important because it gives us a major range to evaluate the rheological models as well as tool-joint corrections. Also, it gives as a result, a better interpretation and validation the study proposed for this research.

# NOMENCLATURE

| | | |
|---|---|---|
| $a$ | = | frictional fractions parameters, dimensionless. |
| $a_1$ | = | transversal area of tool joint, $in^2$. |
| $a_2$ | = | transversal area of pipe, $in^2$. |
| $A$ | = | Robertson and Stiff model parameter similar to $k$, $lbf.sec^B/100\ ft^2$ or $dyne.sec^B/100\ cm^2$ |
| $b$ | = | frictional fractions parameters, dimensionless |
| $B$ | = | Robertson and Stiff model parameter similar to $n$, dimensionless |
| $C$ | = | Robertson and Stiff model correction factor, $1/sec^B$ |
| $C_a$ | = | Casson number, dimensionless |
| $C_c$ | = | Herschel-Bulkley model parameter, dimensionless |
| $C^*_a$ | = | Herschel-Bulkley model parameter, dimensionless |
| $d$ | = | internal diameter of wide pipe, in. |
| $d_{ou}$ | = | internal Diameter of narrow pipe, in. |
| $d_{TJ}$ | = | internal diameter of tool joint, in. |
| $(dp/dL)$ | = | gradient pressure, psi/ft |
| $(dp/dL)_{ds}$ | = | gradient pressure in drill string, psi/ft |
| $(dp/dL)_{TJ}$ | = | gradient pressure in tool joint, psi/ft |
| $D_e$ | = | equivalent Diameter, in. |
| $D_{ea}$ | = | equivalent diameter between two annuli, in. |
| $D_{ep}$ | = | equivalent diameter between two inside pipe diameter, in. |
| $D_N$ | = | nozzle diameter, in. |
| $D_p$ | = | inside pipe Diameter, in. |
| $D_{TJ}$ | = | outside diameter of tool joint, in. |
| $D_1$ | = | outside pipe diameter, in. |
| $D_2$ | = | inside casing diameter, in. |
| $E_{AAP}$ | = | absolute average percent error, % |

| | | |
|---|---|---|
| $f$ | = | friction factor, dimensionless |
| $f_a$ | = | friction factor to the annulus, dimensionless |
| $f_{laminar}$ | = | friction factor to laminar flow, dimensionless |
| $f_p$ | = | friction factor to the pipe, dimensionless |
| $f_{partial}$ | = | Intermediate friction factor (transient and turbulent), dimensionless |
| $f_{transient}$ | = | friction factor to transient flow, dimensionless |
| $f_{turbulent}$ | = | friction factor to turbulent flow, dimensionless |
| $F_c$ | = | contraction mechanical energy loss, lbf ft/lbm |
| $F_e$ | = | enlargement mechanical energy loss, lbf ft/lbm |
| $g_c$ | = | conversion factor, 32.174 lbm ft/lbf $sec^2$ |
| $G$ | = | unified model parameter, dimensionless |
| $k$ | = | consistence index, lbf.$sec^n$/100$ft^2$ or dyne.$sec^n$/100$cm^2$ |
| $k_a$ | = | consistence index in the annulus, lbf.$sec^n$/100$ft^2$ or dyne.$sec^n$/100$cm^2$ |
| $k_p$ | = | consistence index in the pipe, lbf.$sec^n$/100$ft^2$ or dyne.$sec^n$/100$cm^2$ |
| $K_e$ | = | enlargement coefficient, dimensionless |
| $K_c$ | = | contraction coefficient, dimensionless |
| $m$ | = | slope |
| $L$ | = | length, ft |
| $L_{total\ dp}$ | = | total drillpipe length, ft |
| $L_{total\ TJ}$ | = | total tool joint length, ft. |
| $L_1$ | = | length of one tool joint (i.e. pin + box tong length), in |
| $L_2$ | = | length of one drillpipe (without tool joint length), in |
| $n$ | = | flow behavior index, dimensionless |
| $n_a$ | = | flow behavior index in the annulus, dimensionless |
| $n_p$ | = | flow behavior index in the pipe, dimensionless |
| $N$ | = | number of shear rate/shear stress data |

| | | |
|---|---|---|
| $N_{DP}$ | = | numbers of drillpipe |
| $N_{He}$ | = | Hedstrom number, dimensionless |
| $N_{Re}$ | = | Reynolds number, dimensionless |
| $N_{Rec}$ | = | critical value of Reynolds number, dimensionless |
| $N_{TJ}$ | = | number of tool joins, total pipe length/30 ft |
| $p$ | = | Pressure, psi |
| $p_p$ | = | pump pressure, psi |
| $q$ | = | flow rate, gallon/min |
| $q_1$ | = | low flow rate to work for examples, gallon/min |
| $q_2$ | = | high flow rate to work for examples, gallon/min |
| $R$ | = | reading from rheometer |
| $R_3$ | = | reading from rheometer at 3RPM |
| $R_6$ | = | reading from rheometer at 6RPM |
| $R_{600}$ | = | reading from rheometer at 600RPM |
| $R_{300}$ | = | reading from rheometer at 300RPM |
| $v$ | = | average velocity, ft/sec |
| $v_a$ | = | annular average velocity, ft/sec |
| $v_p$ | = | pipe average velocity, ft/sec |
| $V$ | = | velocity from rheometer, RPM |
| $y$ | = | Herschel & Bulkley model parameter, dimensionless |
| $z$ | = | Herschel & Bulkley model parameter, dimensionless |
| $\beta$ | = | ratio of diameters of small to large pipes, dimensionless |
| $\gamma$ | = | shear rate, 1/sec |
| $\gamma_w$ | = | wall shear rate, 1/sec |
| $\gamma^*$ | = | shear rate value corresponding to the geometric mean of the shear stress, $\tau^*$ |
| $\gamma_{min}$ | = | minimum shear stress value of data |
| $\gamma_{max}$ | = | maximum shear stress value of data |

| | | |
|---|---|---|
| $\Delta L$ | = | change in length, ft |
| $\Delta p$ | = | pressure loss, psi |
| $\Delta p_a$ | = | frictional pressure drop in the annulus, psi |
| $\Delta p_b$ | = | frictional pressure loss across the bit, psi |
| $\Delta p_{dc}$ | = | frictional pressure loss inside the drill collars, psi |
| $\Delta p_{dp}$ | = | frictional pressure loss inside the drillpipe, psi |
| $\Delta p_{ds}$ | = | frictional pressure loss inside the drill string, psi |
| $\Delta p_s$ | = | frictional pressure loss in the surface equipment, psi |
| $\Delta p_{TJ}$ | = | tool joint pressure loss, psi |
| $\Delta p_p$ | = | pump pressure loss, psi |
| $(\Delta p_{\text{Total/ 2IDs}})_{ds}$ | = | total pressure drop in the drillstring after 2IDs correction, psi |
| $(\Delta p_{\text{Total 2IDs}})_a$ | = | total pressure drop in the annulus after 2IDs correction, psi |
| $\theta$ | = | angle of divergence on convergence, degrees |
| $\lambda$ | = | Robertson and Stiff model parameter, in. |
| $\mu$ | = | viscosity, cp |
| $\mu_p$ | = | plastic viscosity, cp |
| $\mu_a$ | = | apparent viscosity for Newtonian fluid at 300 $R_{300}$, cp |
| $\mu_e$ | = | equivalent viscosity, cp |
| $\mu_c$ | = | Casson plastic viscosity, lbf.sec/100 $ft^2$ |
| $\rho$ | = | density, Lbm/gal |
| $\tau$ | = | shear stress, lbf/100 $ft^2$ |
| $\tau_c$ | = | Casson yield stress, lbf/100 $ft^2$ |
| $\tau_{\text{calculated}}$ | = | calculated shear stress, lbf/100 $ft^2$ |
| $\tau_{\text{max}}$ | = | Maximum shear stress value of data, lbf/100 $ft^2$ |

$\tau_{measured}$ = measured shear stress, lbf/100 $ft^2$

$\tau_{min}$ = Minimum shear stress value of data, lbf/100 $ft^2$

$\tau_w$ = wall shear stress, lbf/100 $ft^2$

$\tau_y$ = yield point, lbf/100 $ft^2$

$\tau_{yL}$ = lower shear yield point, lbf/100$ft^2$

$\tau_o$ = yield stress, lbf/100 $ft^2$

$\tau^*$ = shear stress value corresponding to the geometric mean of the shear rate, $\gamma^*$

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# APPENDIX A

## Table A1—Rheology and Hydraulics Equations for Newtonian Model.

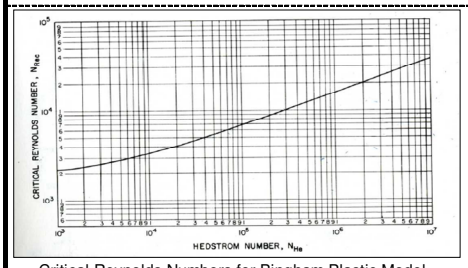
**Hydraulics Equations: NEWTONIAN MODEL,** $\tau = \mu \dot{\gamma}$

| Pipe Flow | Annular Flow |
|---|---|
| $v_p = \frac{0.408\ q}{D_p^{\ 2}}$ ft/sec | $v_a = \frac{0.408 q}{D_2^{\ 2} - D_1^{\ 2}}$ ft/sec |
| $\mu_a = R_{300}$ cp | |
| $N_{\mathrm{Re}} = \frac{928 D_p\ v_p \rho}{\mu_a}$ | $N_{\mathrm{Re}} = \frac{757 (D_2 - D_1) v_a \rho}{\mu_a}$ |
| **Laminar** ($N_{Re}$ < 2,100) $f_p = \frac{16}{N_{\mathrm{Re}}}$ | **Laminar** ($N_{Re}$ < 2,100) $f_a = \frac{16}{N_{\mathrm{Re}}}$ |
| **Turbulent** $f_p = \frac{0.0791}{N_{\mathrm{Re}}^{0.25}}$ | **Turbulent** $f_a = \frac{0.0791}{N_{\mathrm{Re}}^{0.25}}$ |
| $\left(\frac{dp}{dL}\right) = \frac{f_p\ v_p^{\ 2} \rho}{25.81\ D_p}$ psi/ft<br>$\Delta p = \left(\frac{dp}{dL}\right) \Delta L$ psi | $\left(\frac{dp}{dL}\right) = \frac{f_a\ v_a^{\ 2} \rho}{25.81 (D_2 - D_1)}$ psi/ft<br>$\Delta p = \left(\frac{dp}{dL}\right) \Delta L$ psi |
| $\Delta p_{Nozzles}, psi = \frac{156 \rho q^2}{\left(D_{N1}^{\ 2} + D_{N2}^{\ 2} + D_{N3}^{\ 2}\right)^2}$ | |

Marilyn Viloria, April 2006

## Table A2—Rheology and Hydraulics Equations for Bingham Plastics Model.

**Hydraulics Equations: BINGHAM PLASTIC MODEL,** $\tau = \tau_y + \mu_p \dot{\gamma}$

| Pipe Flow | Annular Flow |
|---|---|
| $v_p = \frac{0.408\,q}{D_p^{\,2}}$ ft/sec | $v_a = \frac{0.408q}{D_2^{\,2} - D_1^{\,2}}$ ft/sec |
| $\mu_p = R_{600} - R_{300}$ cp<br>$\tau_y = R_{300} - \mu_p$ lbf/100 ft$^2$ | |
| $\mu_a = \mu_p + \frac{5\tau_y D_p}{v_p}$ cp | $\mu_a = \mu_p + \frac{5\tau_y (D_2 - D_1)}{v_a}$ cp |
| Turbulence Criteria #1<br>$N_{\mathrm{Re}} = \frac{928 D_p v_p \rho}{\mu_a}$<br>($N_{Re}$ < 2,100)→ LAMINAR FLOW<br>$f_p = \frac{16}{N_{\mathrm{Re}}}$ | $N_{\mathrm{Re}} = \frac{757(D_2 - D_1) v_p \rho}{\mu_a}$<br>($N_{Re}$ < 2,100)→ LAMINAR FLOW<br>$f_a = \frac{16}{N_{\mathrm{Re}}}$ |
| Turbulence Criteria #2<br>$He = \frac{37{,}100\ \rho \tau_y D_p^{\,2}}{\mu_{p^2}}$<br>$N_{\mathrm{Re}} = \frac{928 D_p v_p \rho}{\mu p}$ | $D_e = 0.816(D_2 - D_1)$<br>$He = \frac{37{,}100\ \rho \tau_y D_e^{\,2}}{\mu_{p^2}}$<br>$N_{\mathrm{Re}} = \frac{928\ D_e v_a \rho}{\mu p}$ |
|  <br>Critical Reynolds Numbers for Bingham Plastic Model | ($N_{Rep}$ < $N_{Rec}$)<br>LAMINAR FLOW. Use f as criteria #1<br>($N_{Re}$ < $N_{Rec}$)<br>TURBULENT FLOW.Use the following equation:<br>$f_{p,a} = \frac{0.0791}{N_{\mathrm{Re}}^{\,0.25}}$<br>This friction equation is valid for both annular and pipe |
| $\left(\frac{dp}{dL}\right) = \frac{f_p v_p^{\,2} \rho}{25.81\, D_p}$ psi/ft<br>$\Delta p = \left(\frac{dp}{dL}\right) \Delta L$ psi | $\left(\frac{dp}{dL}\right) = \frac{f_a v_a^{\,2} \rho}{25.81 (D_2 - D_1)}$ psi/ft<br>$\Delta p = \left(\frac{dp}{dL}\right) \Delta L$ psi |
| $\Delta p_{Nozzles}, psi = \frac{156 \rho q^2}{\left(D_{N1}^{\,2} + D_{N2}^{\,2} + D_{N3}^{\,2}\right)^2}$ | |

Marilyn Viloria, June 2006

## Table A3—Rheology and Hydraulics Equations for Power Law Model.

**Hydraulics Equations: POWER LAW MODEL,** $\tau = k\,\dot{\gamma}^{\,n}$

| Pipe Flow | Annular Flow |
|---|---|
| $n = 3.32 \log\left(\frac{R_{600}}{R_{300}}\right)$ <br> $k = \frac{510\ R_{300}}{511^{\,n}}$ <br> eq_cps=dyne sec$^n$/100 cm$^2$ | |
| $v_p = \frac{0.408\ q}{D_p^{\,2}}$ ft/sec | $v_a = \frac{0.408\ q}{D_2^{\,2} - D_1^{\,2}}$ ft/sec |
| $N_{Re} = \frac{89100\ \rho v_P^{\,2-n}}{k}\left(\frac{0.0416\ D_P}{3+\frac{1}{n}}\right)^n$ | $N_{Re} = \frac{109000\ \rho v a^{\,2-n}}{k}\left[\frac{0.0208\ (D_2 - D_1)}{2+\frac{1}{n}}\right]^n$ |
| **Laminar** $N_{Re} \le 3470 - 1370\ n$ | |
| $\left(\frac{dp}{dL}\right) = \frac{k v_p^{\,n}\left(\frac{3+1/n}{0.0416}\right)^n}{144000\ D_p^{\,1+n}}$ | $\left(\frac{dp}{dL}\right) = \frac{k v_a^{\,n}\left(\frac{2+1/n}{0.0208}\right)^n}{144000 (D_2 - D_1)^{1+n}}$ |
| **Turbulent** $N_{Re} \ge 4270 - 1370\ n$ | **Turbulent** $N_{Re} \ge 4270 - 1370\ n$ |
| $a = \frac{\log n + 3.93}{50}$ | $b = \frac{1.75 - \log n}{7}$ |
| $f_p = \frac{a}{N_{Re}^{\,b}}$ | $f_a = \frac{a}{N_{Re}^{\,b}}$ |
| $\left(\frac{dp}{dL}\right) = \frac{f_p v_p^{\,2} \rho}{25.81\ D_p}$ psi/ft <br> $\Delta p = \left(\frac{dp}{dL}\right) \Delta L$ psi | $\left(\frac{dp}{dL}\right) = \frac{f_a v_a^{\,2} \rho}{25.81 (D_2 - D_1)}$ psi/ft <br> $\Delta p = \left(\frac{dp}{dL}\right) \Delta L$ psi |
| $\Delta p_{Nozzles} = \frac{156 \rho q^2}{\left(D_{N1}^{\,2} + D_{N2}^{\,2} + D_{N3}^{\,2}\right)^2}$ psi | |

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## Table A4—Rheology and Hydraulics Equations for API RP13D Model.

**Hydraulics Equations: API RP 13D,** $\tau = k \dot{\gamma}^{n}$

| Pipe Flow | Annular Flow |
|---|---|
| $n_p = 3.32 \log\left(\frac{R_{600}}{R_{300}}\right)$ | $n_a = 0.657 \log\left(\frac{R_{100}}{R_3}\right)$ |
| $k_p = \frac{5.11\ R_{600}}{1{,}022^{\,n_p}}$ dyne sec$^n$/cm$^2$ | $k_a = \frac{5.11\ R_{100}}{170.2^{\,n_a}}$ dyne sec$^n$/cm$^2$ |
| $v_p = \frac{0.408\ q}{D_p^{\,2}}$ ft/sec | $v_a = \frac{0.408\ q}{D_2^{\,2} - D_1^{\,2}}$ ft/sec |
| $\mu_e = 100 k_p \left(\frac{96 v_p}{D_p}\right)^{n_p - 1} \left(\frac{3n_p + 1}{4n_p}\right)^{n_p}$ cp | $\mu_e = 100 k_a \left(\frac{144 v_a}{D_2 - D_1}\right)^{n_a - 1} \left(\frac{2n_a + 1}{3n_a}\right)^{n_a}$ cp |
| $N_{\mathrm{Re}} = \frac{928\, D\, v_p \rho}{\mu_e}$ | $N_{\mathrm{Re}} = \frac{928 (D_2 - D_1) v_a \rho}{\mu_e}$ |
| **Laminar** ($N_{Re}$ < 2,100) $f_p = \frac{16}{N_{\mathrm{Re}}}$ | **Laminar** ($N_{Re}$ < 2,100) $f_a = \frac{24}{N_{\mathrm{Re}}}$ |
| **Turbulent** $a = \frac{\log n_p + 3.93}{50}$, $b = \frac{1.75 - \log n_p}{7}$, $f_p = \frac{a}{N_{\mathrm{Re}}^{\,b}}$ | **Turbulent** $a = \frac{\log n_a + 3.93}{50}$, $b = \frac{1.75 - \log n_a}{7}$, $f_a = \frac{a}{N_{\mathrm{Re}}^{\,b}}$ |
| $\left(\frac{dp}{dL}\right) = \frac{f_p v_p^{\,2} \rho}{25.81 D_p}$ psi/ft; $\Delta p = \left(\frac{dp}{dL}\right) \Delta L$ psi | $\left(\frac{dp}{dL}\right) = \frac{f_a v_a^{\,2} \rho}{25.81 (D_2 - D_1)}$ psi/ft; $\Delta p = \left(\frac{dp}{dL}\right) \Delta L$ psi |
| $\Delta p_{Nozzles} = \frac{156 \rho q^2}{\left(D_{N1}^{\,2} + D_{N2}^{\,2} + D_{N3}^{\,2}\right)^2}$ psi | |

*HCJ May 30, 2002*

# Table A5—Rheology and Hydraulics Equations for Herschel-Bulkley Model.

## Hydraulics Equations: HERSCHEL-BULKLEY MODEL, $\tau = \tau_0 + k\dot{\gamma}^n$

| Pipe Flow | Annular Flow |
|---|---|
| $\tau_0 = \dfrac{\tau^{*2} - \tau_{\min}\tau_{\max}}{2\tau^* - \tau_{\min} - \tau_{\max}}$ | τo=lbf/100sq ft |
| where τ* is the shear stress value corresponding to the geometric mean of the shear rate,γ* | |
| $\gamma^* = \sqrt{\gamma_{\min}\gamma_{\max}}$ | γ=1/sec |
| $\log(\tau-\tau_o)=\log(k)+n\log(\gamma)$ $\quad n=\dfrac{\sum\log(\tau-\tau_o)\sum\log(\gamma)-N\sum(\log(\tau-\tau_0)\log(\gamma))}{(\sum\log\gamma)^2-N\sum(\log\gamma)^2}$ | n=DIMENSIONLESS |
| $\log(k)=\dfrac{\sum\log(\tau-\tau_o)-n\sum\log(\gamma)}{N}$ | N=data number |
| | k=lbf*sec$^n$/100sq ft |
| $v_p = \dfrac{0.408\,q}{D_p^{\,2}}$ ft/sec | $v_a = \dfrac{0.408\,q}{D_2^{\,2} - D_1^{\,2}}$ ft/sec |
| $N_{\mathrm{Re}} = \left[\dfrac{2(3n+1)}{n}\right]\left\{\dfrac{\rho v_p^{(2-n)}\left(\frac{D_p}{2}\right)^n}{\tau_o\left(\frac{D_p}{2v_p}\right)^n + k\left[\frac{(3n+1)}{nC_c}\right]^n}\right\}$ | $N_{\mathrm{Re}} = \left[\dfrac{4(2n+1)}{n}\right]\left\{\dfrac{\rho v_a^{(2-n)}\left(\frac{D_2-D_1}{2}\right)^n}{\tau_o\left(\frac{D_2-D_1}{2v_a}\right)^n + k\left[\frac{2(2n+1)}{nC_a^*}\right]^n}\right\}$ D=ft ρ=lbm/ft$^3$ |
| $N_{\mathrm{Re}\,c} = \left[\dfrac{4(3n+1)}{ny}\right]^{\frac{1}{1-z}}$ | $N_{\mathrm{Re}\,c} = \left[\dfrac{8(2n+1)}{ny}\right]^{\frac{1}{1-z}}$ |
| $y = \dfrac{\log n + 3.93}{50}$ | $z = \dfrac{1.75 - \log n}{7}$ |
| **Laminar** $N_{\mathrm{RE}} < N_{\mathrm{RE}\,C}$ | |
| $\left(\dfrac{dp}{dL}\right) = \dfrac{4k}{144000D_p}\left\{\left(\dfrac{\tau_0}{k}\right) + \left[\left(\dfrac{3n+1}{nC_c}\right)\left(\dfrac{8q}{\pi D_p^{\,3}}\right)\right]^n\right\}$ | $\left(\dfrac{dp}{dL}\right) = \dfrac{4k}{144000(D_2-D_1)}\left\{\left(\dfrac{\tau_0}{k}\right) + \left[\left[\dfrac{16(2n+1)}{nC_a^*(D_2-D_1)}\right]\left[\dfrac{q}{\pi(D_2^2-D_1^2)}\right]\right]^n\right\}$ q= ft$^3$/s dp/dL=psi/ft k=lbf*sec$^n$/100ft$^2$ |
| **Turbulent** $N_{\mathrm{Re}} > N_{\mathrm{Re}\,c}$ | |
| $f_p = y\left(C_c N_{\mathrm{Re}}\right)^{-z}$ | $f_a = y\left(C_a^* N_{\mathrm{Re}}\right)^{-z}$ |
| $Cc = 1 - \left(\dfrac{1}{2n+1}\right)\dfrac{\tau_0}{\tau_0 + k\left[\frac{(3n+1)q}{n\pi(D_p/2)^3}\right]^n}$ | $C_a^* = 1 - \left(\dfrac{1}{n+1}\right)\dfrac{\tau_0}{\tau_0 + k\left\{\left[\frac{2(2n+1)}{n((D_2/2)-(D_1/2))}\right]\left[\frac{q}{\pi((D_2/2)^2-(D_1/2)^2)}\right]\right\}^n}$ |
| $\left(\dfrac{dp}{dL}\right) = \dfrac{f_p\,q^2\rho}{144\,\pi^2 D_p^{\,5}}$ psi/ft | $\left(\dfrac{dp}{dL}\right) = \dfrac{f_a\,q^2\rho}{144\,\pi^2(D_2-D_1)(D_2^2-D_1^2)^2}$ psi/ft |
| $\Delta p = \left(\dfrac{dp}{dL}\right)\Delta L$ psi | $\Delta p = \left(\dfrac{dp}{dL}\right)\Delta L$ psi |
| $\Delta p_{Nozzles}, psi = \dfrac{156\rho q^2}{\left(D_{N1}^{\,2} + D_{N2}^{\,2} + D_{N3}^{\,2}\right)^2}$ | |

Marilyn Viloria, June 2006

## Table A6—Rheology and Hydraulics Equations for Unified Model.

**Hydraulics Equations:Unified Rheological Model,** $\tau = \tau_0 + k\dot{\gamma}^n$

| Pipe Flow | Annular Flow |
|---|---|
| $\mu_p = R_{600} - R_{300}$ $\quad \tau_y = R_{300} - \mu_p$ | $\tau_o = 1.066\,(2R_3 - R_6)$ $\quad \mu_p$=cp, $\tau$= lbf/100ft$^2$ |
| $n_p = 3.32 \log\left(\frac{2\mu_p + \tau_y}{\mu_p + \tau_y}\right)$ <br> $k_p = 1.066\left(\frac{\mu_p + \tau_y}{511^{n_p}}\right)$ | $n_a = 3.32\log\left(\frac{2\mu_p + \tau_y - \tau_o}{\mu_p + \tau_y - \tau_o}\right)$ <br> $k_a = 1.066\left(\frac{\mu_p + \tau_y - \tau_o}{511^{na}}\right)$ <br> k=lbf*sec$^n$/100 ft$^2$ |
| $G = \left(\frac{(3-\alpha)n+1}{(4-\alpha)n}\right)\left(1+\frac{\alpha}{2}\right)$ <br> α=1 for annuli, α=0 for pipe | |
| $v_p = \frac{24.51\,q}{D^2}$ | $v_a = \frac{24.51\,q}{D_2^2 - D_1^2}$ <br> v=ft/min |
| $\gamma_w = \frac{1.6 * G * v}{D_e}$ | $\gamma_w = \sec^{-1}$ |
| $\tau_w = \left[\left(\frac{4-\alpha}{3-\alpha}\right)^n \tau_o + k\gamma_w^n\right]$ <br> $\tau_w$= lbf/100ft$^2$ | |
| $N_{Re_p} = \frac{\rho v_p^2}{19.36\,\tau_w}$ | $N_{Re_a} = \frac{\rho v_a^2}{19.36\,\tau_w}$ |
| **Laminar:** $f_{laminar} = \frac{16}{N_{Re}}$ <br> **Transient:** $f_{transient} = \frac{16N_{Re}}{(3470-1370n_p)^2}$ <br> **Turbulent:** $a = \frac{\log n + 3.93}{50}$, $b = \frac{1.75 - \log n}{7}$, $f_{turbulent} = \frac{a}{N_{Re}^b}$ | **Laminar:** $f_{laminar} = \frac{24}{N_{Re}}$ <br> **Transient:** $f_{transient} = \frac{16N_{Re}}{(3470-1370n_a)^2}$ <br> **Turbulent:** $a = \frac{\log n + 3.93}{50}$, $b = \frac{1.75 - \log n}{7}$, $f_{turbulent} = \frac{a}{N_{Re}^b}$ |
| $f_{partial} = \left(f_{transient}^{-8} + f_{turbuelent}^{-8}\right)^{-1/8}$ | |
| $f_p = \left(f_{partial}^{12} + f_{laminal}^{12}\right)^{1/12}$ | $f_a = \left(f_{partial}^{12} + f_{laminal}^{12}\right)^{1/12}$ |
| $\left(\frac{dp}{dL}\right) = 1.076\,\frac{f_p v_p^2 \rho}{10^5 D_p}$ psi/ft <br> $\Delta p = \left(\frac{dp}{dL}\right)\Delta L$ psi | $\left(\frac{dp}{dL}\right) = \frac{1.076\,f_a v_a^2 \rho}{10^5 D_e}$ psi/ft <br> $\Delta p = \left(\frac{dp}{dL}\right)\Delta L$ psi |
| $\Delta p_{Nozzles} = \frac{156\rho q^2}{\left(D_{N1}^2 + D_{N2}^2 + D_{N3}^2\right)^2}$ psi | |

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## Table A7—Rheology and Hydraulics Equations for Robertson and Stiff Model "Original".

**Hydraulics Equations:ROBERTSON AND STIFF MODEL"Original",** $\tau = A(\dot{\gamma} + C)^B$

| Pipe Flow | Annular Flow |
|---|---|
| $C = \frac{\gamma_{min} * \gamma_{max} - \gamma^{*2}}{2\gamma^{*} - \gamma_{min} - \gamma_{max}}$ 1/sec$^B$ | |
| $\log(\tau) = \log(A) + B\log(\gamma)$ { $B = \frac{\sum \log\tau . \sum \log(\gamma + C) - N . \sum (\log\tau . \log(\gamma + C))}{\left[\log(\gamma + C)^2 - N . \sum \log(\gamma + C)\right]^2}$ DIMENSIONLESS; $\log(A) = \frac{\sum \log\tau - B . \sum \log(\gamma + C)}{N}$ lbf*sec$^B$/100ft$^2$ | |
| $v_p = \frac{0.408\ q}{D_p^{\ 2}}$ ft/sec | $v_a = \frac{0.408\ q}{D_2^{\ 2} - D_1^{\ 2}}$ ft/sec |
| $N_{RE} = \frac{89100\ \rho {v_p}^{2-B}}{A}\left(\frac{0.416\ D_p}{3 + \frac{1}{B}}\right)^B$ | $N_{RE} = \frac{109000\ \rho v a^{2-B}}{A}\left[\frac{0.0208\ (D_2 - D_1)}{2 + \frac{1}{B}}\right]^B$ v=ft/sec; A=dyne sec$^B$/100cm$^2$=eq cp |
| **Laminar** $N_{RE} \le 3470 - 1370B$ | **Laminar** $N_{RE} \le 3470 - 1370B$ |
| $\left(\frac{dp}{dL}\right) = 8.33E-4 \times 2^{2+B} \times A\left\{\left(\frac{1+3B}{B}\right)\left[\frac{0.2 v_p + \frac{C}{6} D_p}{D_p^{\left(\frac{1+B}{B}\right)}}\right]\right\}^B$ | $\left(\frac{dp}{dL}\right) = 8.33E-4 \times 4^{1+B} \times A\left\{\left(\frac{1+2B}{B}\right)\left[\frac{0.2 v_a + \frac{C}{8}(D_2 - D_1)}{(D_2 - D_1)^{\left(\frac{1+B}{B}\right)}}\right]\right\}^B$ dP/dL=psi/ft; D=in; v=ft/min; A=lbf*sec$^B$/100sq ft |
| **Turbulent** $N_{Re} \ge 4270 - 1370\ B$ | |
| $a = \frac{\log B + 3.93}{50}$ | $b = \frac{1.75 - \log B}{7}$ |
| $f_p = \frac{a}{N_{Re}^{\ b}}$ | $f_a = \frac{a}{N_{Re}^{\ b}}$ |
| $\left(\frac{dp}{dL}\right) = \frac{f_p {v_p}^2 \rho}{25.81 D_p}$ psi/ft | $\left(\frac{dp}{dL}\right) = \frac{f_a {v_a}^2 \rho}{25.81 (D_2 - D_1)}$ psi/ft |
| $\Delta p = \left(\frac{dp}{dL}\right) \Delta L$ psi | $\Delta p = \left(\frac{dp}{dL}\right) \Delta L$ psi |
| $\Delta p_{Nozzles}, psi = \frac{156 \rho q^2}{\left(D_{N1}^{\ 2} + D_{N2}^{\ 2} + D_{N3}^{\ 2}\right)^2}$ Marilyn Viloria, June 2006 | |

## Table A8—Rheology and Hydraulics Equations for Robertson and Stiff Model "Yield Point".

**Hydraulics Equations:ROBERTSON-STIFF MODEL"YIELD POINT",** $\tau = A(\dot{\gamma} + C)^B$

| Pipe Flow | Annular Flow |
|---|---|
| $C = \dfrac{\gamma_{min} * \gamma_{max} - \gamma^{*2}}{2\gamma^{*} - \gamma_{min} - \gamma_{max}}$ 1/sec$^B$ | |
| $\log(\tau) = \log(A) + B\log(\gamma)$ { $B = \dfrac{\sum\log\tau.\sum\log(\gamma+C) - N.\sum(\log\tau.\log(\gamma+C))}{\left[\log(\gamma+C)^2 - N.\sum\log(\gamma+C)\right]^2}$ DIMENSIONLESS; $\log(A) = \dfrac{\sum\log\tau - B.\sum\log(\gamma+C)}{N}$ lbf*sec$^B$/100ft$^2$ | |
| $v_p = \dfrac{0.408\ q}{D_p^{\ 2}}$ ft/sec | $v_a = \dfrac{0.408\ q}{D_2^{\ 2} - D_1^{\ 2}}$ ft/sec |
| $N_{Re} = \dfrac{89100\ \rho {v_P}^{2-B}}{A}\left(\dfrac{0.416\ D_P}{3+\frac{1}{B}}\right)^B$ | $N_{Re} = \dfrac{109000\ \rho {v_a}^{2-B}}{A}\left[\dfrac{0.0208\ (D_2 - D_1)}{2+\frac{1}{B}}\right]^B$ A=dyne sec$^B$/100cm$^2$=eq cp |
| **Laminar** $N_{Re} \le 3470 - 1370\ B$ | |
| $\lambda = \dfrac{2(AC^B)}{(dp/dL)}$ NOTE: that ÷ is grater than zero if the fluid has a yield point(÷o=AC$^B$) | $\lambda = \dfrac{AC^B}{(dp/dL)}$ C=1/sec$^B$ dP/dL=lbf/ft$^2$/ft A=lbf.sec$^B$/ft$^2$ |
| Use same equation for annular or pipe in laminar flow, D(ft)=$D_2$-$D_1$, annulus. D(ft)=$D_p$, pipe. | $q = \pi\left\{\left[\dfrac{1}{2A}\left(\dfrac{dp}{dL}\right)\right]^{\frac{1}{B}}\left(\dfrac{B}{3B+1}\right)\left[\left(\dfrac{D}{2}\right)^{\frac{3B+1}{B}} - (\lambda)^{\frac{3B+1}{B}}\right] - \dfrac{1}{3}C\left[\left(\dfrac{D}{2}\right)^3 - (\lambda)^3\right]\right\}$ NOTE: Apply solve from EXCEL to find a solution for pressure loss q=ft$^3$/sec ÷P/L=lbf/ft$^2$/ft A=lbf*sec$^B$/sq ft |
| **Turbulent** $N_{Re} \ge 4270 - 1370\ B$ | |
| $a = \dfrac{\log B + 3.93}{50}$ | $b = \dfrac{1.75 - \log B}{7}$ |
| $f_p = \dfrac{a}{N_{Re}^{\ b}}$ | $f_a = \dfrac{a}{N_{Re}^{\ b}}$ |
| $\left(\dfrac{dp}{dL}\right) = \dfrac{f_p {v_p}^2 \rho}{25.81\ D}$ psi/ft | $\left(\dfrac{dp}{dL}\right) = \dfrac{f_a {v_a}^2 \rho}{25.81(D_2 - D_1)}$ psi/ft |
| $\Delta p = \left(\dfrac{dp}{dL}\right)\Delta L$ psi | $\Delta p = \left(\dfrac{dp}{dL}\right)\Delta L$ psi |
| $\Delta p_{Nozzles}, psi = \dfrac{156\rho q^2}{\left(D_{N1}^{\ 2} + D_{N2}^{\ 2} + D_{N3}^{\ 2}\right)^2}$ Marilyn Viloria, June 2006 | |

## Table A9—Rheology and Hydraulics Equations for Casson.

**Hydraulics Equations: CASSON MODEL,** $\tau^{1/2} = \tau_c^{1/2} + \mu_c^{1/2}\dot{\gamma}^{1/2}$

| Pipe Flow | Annular Flow |
|---|---|
| $v_p = \frac{0.408\ q}{D_p^{\ 2}}$ ft/sec | $v_a = \frac{0.408 q}{D_2^{\ 2} - D_1^{\ 2}}$ ft/sec |

$$\tau^{1/2} = \tau_c^{1/2} + \mu_c^{1/2}\dot{\gamma}^{1/2} \left\{ \begin{array}{l} \mu_c^{1/2} = \dfrac{\sum\log(\tau^{1/2})\sum\log(\gamma^{1/2}) - N\sum\left(\log(\tau^{1/2})\log(\gamma^{1/2})\right)}{(\sum\log\gamma^{1/2})^2 - N\sum(\log\gamma^{1/2})^2} \\ \tau_c^{1/2} = \dfrac{\sum\log(\tau^{1/2}) - \mu_c^{1/2}\sum\log(\gamma^{1/2})}{N} \end{array} \right.$$

| | |
|---|---|
| $N_{\mathrm{Re}} = \frac{928\, D\, v_p \rho}{\mu_c}$ | $N_{\mathrm{Re}} = \frac{757\,(D_2 - D_1)\, v_a \rho}{\mu_c}$ |

**Turbulence Criteria**

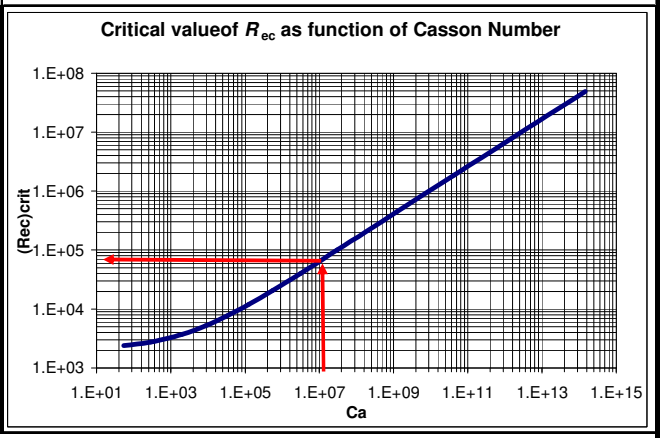
1.-Calculate the Casson number, for annulus and pipe.

$$Ca = \frac{D^2 \tau_c \rho}{32.174\ \mu_c^{\ 2}}$$

2.- Estimate from figure the $N_{RE}$ critical, for annulus and pipe

3- Compare $N_{RE}$ critical with $N_{RE}$

4- Use same equation for annular or pipe in laminar flow,
Use: D=D2-D1, annulus
D=Dp, pipe




**Laminar**

$$q = \frac{\pi D^3}{8\mu_c}\left[\frac{D\left(\frac{dp}{dL}\right)}{16} - \frac{4}{7}\sqrt{\tau_c}\sqrt{\frac{(dp/dL)D}{4}} - \frac{64\,\tau_c^{\ 4}}{84\left(\frac{dp}{dL}\right)D^3} + \frac{\tau_c}{3}\right]$$

dp/dL=lbf/ft$^2$/ft

NOTE: Apply solve from EXCEL to find a solution for pressure loss

| | |
|---|---|
| **Turbulent** $f_p = \frac{0.0791}{N_{\mathrm{Re}}^{\ 0.25}}$ | **Turbulent** $f_a = \frac{0.0791}{N_{\mathrm{Re}}^{\ 0.25}}$ |
| $\left(\frac{dp}{dL}\right) = \frac{f_p\, v_p^{\ 2}\, \rho}{25.81\, D_p}$ psi/ft | $\left(\frac{dp}{dL}\right) = \frac{f_a\, v_a^{\ 2}\, \rho}{25.81\,(D_2 - D_1)}$ psi/ft |
| $\Delta p = \left(\frac{dp}{dL}\right)\Delta L$ psi | $\Delta p = \left(\frac{dp}{dL}\right)\Delta L$ psi |

$$\Delta p_{Nozzles}, psi = \frac{156\rho q^2}{\left(D_{N1}^{\ 2} + D_{N2}^{\ 2} + D_{N3}^{\ 2}\right)^2}$$

Marilyn Viloria, June 2006

# APPENDIX B

***Example:***

Drillpipe-5 in. 19.5 S-135 w/4.5 IF (6.75in.x 3in. connection): $D_1$= 5 in, $D_p$ =4.5 in

Casing 11 7/8 in.x10.711 in., $D_2$=10.711 in.

Length of well= 12440 ft

Rheological data= same as in chapter IV

$q_1$=100 GPM, $q_2$=665 GPM

Density (ρ) = 11.55 lb/galm

Bit: 10 5/8 in. w/3: 28/32 in. jets

$\Delta p_s$=0

## B-1 NEWTONIAN FLUID

### $q_1$=100 gallon/min

- Pipe Flow

$$v_p = \frac{0.408q}{D_p{}^2} = \frac{0.408\times 100}{(4.5)^2} = 2.015 \text{ ft/sec}$$

$\mu_a$= $R_{300}$= 58 cp

$$N_{\text{Re}} = \frac{928 D_p v_p \rho}{\mu_a} = \frac{928\times 4.5\times 2.015\times 11.55}{58} = 1675.520$$

If $N_{\text{Re}} < N_{\text{Rec}}$ →flow is laminar.

$f$= 16/ $N_{\text{Re}}$ = 16/ 1675.52=0.00955

$$\left(\frac{dp}{dL}\right) = \frac{f v_p{}^2 \rho}{25.81 D_p} = \frac{0.00955\times (2.0125)^2 \times 11.55}{25.81\times 4.5} = 0.0039 \text{ psi/ft}$$

$$\Delta p_{ds} = \left(\frac{dp}{dL}\right)\Delta L = 0.0039\times 12440 = 47.846 \text{ psi}$$

- Annular Flow

$$v_a = \frac{0.408q}{(D_2{}^2 - D_1^2)} = \frac{0.408\times 100}{(10.711^2 - 5^2)} = 0.4547 \text{ ft/sec}$$

$$N_{\text{Re}} = \frac{757(D_2 - D_1)v_a\rho}{\mu_a} = \frac{757\times(10.711-5)\times 0.4547\times 11.55}{58} = 391.477$$

If $N_{\text{Re}} < N_{\text{Rec}} \rightarrow$ flow is laminar.

$f = 16/ N_{\text{Re}} = 16/ 391.477 = 0.04087$

$$\left(\frac{dp}{dL}\right) = \frac{fv_a{}^2\rho}{25.81(D_2 - D_1)} = \frac{0.04087\times(0.4547)^2\times 11.55}{25.81\times(10.711-5)} = 0.00066 \quad \text{psi/ft}$$

$$\Delta p_a = \left(\frac{dp}{dL}\right)\Delta L = 0.00066\times 12440 = 8.237 \text{psi}$$

- Frictional pressure losses across the bit, $\Delta p_b$:

$$\Delta p_b = \frac{156\rho q^2}{(D_{N1}{}^2 + D_{N2}{}^2 + D_{N3}{}^2)^2} = \frac{156\times 11.55\times 100^2}{(28^2 + 28^2 + 28^2)^2} = 3.257 \text{ psi}$$

Finally, the pump pressure:

$\Delta p_p = \Delta p_s + \Delta p_{ds} + \Delta p_b + \Delta p_a = 0+47.846+3.257+8.237 = 59.340$ psi

**$q_2$=665 gallon/min**

- Pipe Flow

$$v_p = \frac{0.408q}{D_p{}^2} = \frac{0.408\times 665}{(4.5)^2} = 13.399 \text{ ft/sec}$$

$\mu_a$=58 cp

$$N_{\text{Re}} = \frac{928D_p v_p\rho}{\mu_a} = \frac{928\times 4.5\times 13.399\times 11.55}{58} = 11142.208$$

If $N_{\text{Re}} > N_{\text{Rec}} \rightarrow$flow is turbulent.

$f = 0.0791/ N_{\text{Re}}{}^{0.25} = 0.0791/ (11142.208)^{0.25} = 0.00769$

$$\left(\frac{dp}{dL}\right)=\frac{fv_p{}^2\rho}{25.81D_p}=\frac{0.00769\times(13.399)^2\times11.55}{25.81\times4.5}$$ =0.1375 psi/ft

$$\Delta p_{ds}=\left(\frac{dp}{dL}\right)\Delta L=0.1375\times12440=1709.94 \quad \text{psi}$$

- Annular Flow

$$v_a=\frac{0.408q}{(D_2{}^2-D_1^2)}=\frac{0.408\times665}{(10.711^2-5^2)}$$ =3.024 ft/sec

$$N_{\mathrm{Re}}=\frac{757(D_2-D_1)v_a\rho}{\mu_a}=\frac{757\times(10.711-5)\times3.024\times11.55}{58}$$ =2603.322

If $N_{\mathrm{Re}} > N_{\mathrm{Re}c} \rightarrow$flow is turbulent.

$f$= 0.0791/ $N_{\mathrm{Re}}{}^{0.25}$= 0.0791/ (2603.322)$^{0.25}$=0.01107

$$\left(\frac{dp}{dL}\right)=\frac{fv_a{}^2\rho}{25.81(D_2-D_1)}=\frac{0.01107\times(3.024)^2\times11.55}{25.81\times(10.711-5)}=0.00793 \text{ psi/ft}$$

$$\Delta p_a=\left(\frac{dp}{dL}\right)\Delta L=0.00793\times12440=98.710 \text{ psi}$$

- Frictional pressure losses across the bit, $\Delta p_b$:

$$\Delta p_b=\frac{156\rho q^2}{(D_{N1}{}^2+D_{N2}{}^2+D_{N3}{}^2)^2}=\frac{156\times11.5\times665^2}{(28^2+28^2+28^2)^2}=144.037 \text{ psi}$$

Finally, the pump pressure:

$\Delta p_p = \Delta p_s + \Delta p_{ds} + \Delta p_b + \Delta p_a$=0+1709.94+144.037+98.710=1952.687 psi

## B-2 BINGHAM PLASTIC FLOW

### $q_1$=100 gallon/min

- Pipe Flow

$$v_p=\frac{0.408q}{D_p{}^2}=\frac{0.408\times100}{(4.5)^2}=2.015 \text{ ft/sec}$$

$\mu_p = R_{600} - R_{300}$=92-58= 34 cp

$\tau_y = R_{300} - \mu_p$ = 58-34= 24 lbf/100 ft$^2$

$$\mu_a = \mu_p + \frac{5\tau_y D_p}{v_p} = 34 + \frac{5\times 24\times 4.5}{2.015} = 301.990 cp$$

$$N_{\mathrm{Re}} = \frac{928 D_p v_p \rho}{\mu_p} = \frac{928\times 4.5\times 2.015\times 11.55}{301.990} = 321.829$$

If $N_{\mathrm{Re}} < N_{\mathrm{Rec}} \rightarrow$flow is laminar.

$f$= 16/ $N_{\mathrm{Re}}$ = 16/ 321.829=0.04972

$$\left(\frac{dp}{dL}\right) = \frac{f v_p{}^2 \rho}{25.81 D_p} = \frac{0.04972\times (2.015)^2\times 11.55}{25.81\times 4.5} = 0.0195 \text{ psi/ft}$$

$$\Delta p_{ds} = \left(\frac{dp}{dL}\right)\Delta L = 0.0195\times 12440 = 249.7 \text{ psi}$$

- Annular Flow

$$v_a = \frac{0.408 q}{(D_2{}^2 - D_1^2)} = \frac{0.408\times 100}{(10.711^2 - 5^2)} = 0.4547 \text{ ft/sec}$$

$$\mu_a = \mu_p + \frac{5\tau_y (D_2 - D_1)}{v_a} = 34 + \frac{5\times 24\times (10.711-5)}{0.4547} = 1541.192 \text{ cp}$$

$$N_{\mathrm{Re}} = \frac{757 (D_2 - D_1) v_a \rho}{\mu_a} = \frac{757\times (10.711-5)\times 0.4547\times 11.55}{1541.192} = 14.732$$

If $N_{\mathrm{Re}} < N_{\mathrm{Rec}} \rightarrow$flow is laminar.

$f$= 16/ $N_{\mathrm{Re}}$ = 16/ 14.732=1.08608

$$\left(\frac{dp}{dL}\right) = \frac{f v_a{}^2 \rho}{25.81 (D_2 - D_1)} = \frac{1.08608\times (0.4547)^2\times 11.55}{25.81\times (10.711-5)} = 0.01759 psi/ft$$

$$\Delta p_a = \left(\frac{dp}{dL}\right)\Delta L = 0.01759\times 12440 = 218.8839 psi$$

- Frictional pressure losses across the bit, $\Delta p_b$:

$$\Delta p_b = \frac{156\rho q^2}{(D_{N1}{}^2 + D_{N2}{}^2 + D_{N3}{}^2)^2} = \frac{156\times 11.5\times 100^2}{(28^2+28^2+28^2)^2} = 3.257 psi$$

Finally, the pump pressure:

$\Delta p_{p=} \Delta p_s + \Delta p_{ds} + \Delta p_b + \Delta p_a$=0+249.7+3.257+218.88=471.84 psi

**$q_2$=665 gallon/min**

- Pipe Flow

$$v_p = \frac{0.408q}{D_p{}^2} = \frac{0.408\times 665}{(4.5)^2} = 13.399 \text{ ft/sec}$$

$\mu_p$=34 cp

$\tau_y$= 24 lbf/100 ft$^2$

$$\mu_a = \mu_p + \frac{5\tau_y D_p}{v_p} = 34 + \frac{5\times 24\times 4.5}{13.399} = 74.302 cp$$

$$N_{\text{Re}} = \frac{928 D_p v_p \rho}{\mu_p} = \frac{928\times 4.5\times 13.399\times 11.55}{74.302} = 8697.9956$$

If $N_{\text{Re}} > N_{\text{Re}c} \rightarrow$flow is turbulent.

$f$= 0.0791/ $N_{\text{Re}}{}^{0.25}$= 0.0791/ (8697.9956)$^{0.25}$=0.00819

$$\left(\frac{dp}{dL}\right) = \frac{f v_p{}^2 \rho}{25.81 D_p} = \frac{0.00819\times (13.399)^2\times 11.55}{25.81\times 4.5} = 0.14623 \text{ psi/ft}$$

$$\Delta p_{ds} = \left(\frac{dp}{dL}\right)\Delta L = 0.14623\times 12440 = 1819.15123 \text{ psi}$$

- Annular Flow

$$v_a = \frac{0.408q}{(D_2{}^2 - D_1^2)} = \frac{0.408\times 665}{(10.711^2 - 5^2)} = 3.024 \text{ ft/sec}$$

$$\mu_a = \mu_p + \frac{5\tau_y (D_2 - D_1)}{v_a} = 34 + \frac{5 \times 24 \times (10.711 - 5)}{3.024} = 260.627 cp$$

$$N_{\mathrm{Re}} = \frac{757(D_2 - D_1) v_a \rho}{\mu_a} = \frac{757 \times (10.711 - 5) \times 3.024 \times 11.55}{260.627} = 579.365$$

If $N_{\mathrm{Re}} < N_{\mathrm{Rec}} \rightarrow$ *flow is laminar.*

$f = 16 / N_{\mathrm{Re}} = 16 / (579.365) = 0.02762$

$$\left(\frac{dp}{dL}\right) = \frac{f v_a{}^2 \rho}{25.81(D_2 - D_1)} = \frac{0.02762 \times (3.024)^2 \times 11.55}{25.81 \times (10.711 - 5)} = 0.01979 \, psi/ft$$

$$\Delta p_a = \left(\frac{dp}{dL}\right) \Delta L = 0.01979 \times 12440 = 246.169 \, \mathrm{psi}$$

- Frictional pressure losses across the bit, $\Delta p_b$:

$$\Delta p_b = \frac{156 \rho q^2}{(D_{N1}{}^2 + D_{N2}{}^2 + D_{N3}{}^2)^2} = \frac{156 \times 11.5 \times 665^2}{(28^2 + 28^2 + 28^2)^2} = 144.037 \, \mathrm{psi}$$

Finally, the pump pressure:

$\Delta p_p = \Delta p_s + \Delta p_{ds} + \Delta p_b + \Delta p_a = 0 + 1819.15 + 144.037 + 246.169 = 2209.356$ psi

## B-3 POWER LAW FLOW

### $q_1$=100 gallon/min

- Pipe Flow

a. Velocity:

$$v_p = \frac{0.408q}{D_p{}^2} = \frac{0.408 \times 100}{(4.5)^2} = 2.015 \text{ ft/sec}$$

b. Reynolds number:

$$n = 3.32 \log\left(\frac{R_{600}}{R_{300}}\right) = 3.32 \log\left(\frac{92}{58}\right) = 0.6652$$

$$k = \frac{510R_{300}}{511^{n}} = \frac{510\times 58}{511^{0.6652}} = 467.058 \quad \text{dyne.sec}^{n}/100\text{cm}^{2}$$

$$N_{\text{Re}} = \frac{89100 v_p^{\,2-n}\rho}{k}\left(\frac{0.0416 D_p}{3+\frac{1}{n}}\right)^{n}$$

$$N_{\text{Re}} = \frac{89100\times 2.015^{2-0.6652}\times 11.55}{467.068}\left(\frac{0.0416\times 4.5}{3+\frac{1}{0.6652}}\right)^{0.6652} = 676.774$$

c. For laminar flow, critical value

$N_{Rec}$ = 3470-1370$n$= 3470-1370x0.6652=2558.7

For turbulent flow, critical value

$N_{Rec}$ = 4270-1370$n$= 4270-1370x0.6652=3358.7

d. Regime flow determination:

Comparison between $N_{\text{Re}}$ and $N_{Rec}$

If $N_{\text{Re}} < N_{Rec} \rightarrow$flow is laminar.

Friction factor is included in Eq.5.21.

e. - Frictional pressure loss calculation inside drillstring:

*Laminar:*

$$\left(\frac{dp}{dL}\right) = \frac{k v_p^{n}\left(\frac{3+1/n}{0.0416}\right)^{n}}{144000 D_p^{1+n}} = \frac{467.058\times 2.015^{0.6652}\times\left(\frac{3+1/0.6652}{0.0416}\right)^{0.6652}}{144000\times 4.5^{1+0.6652}}$$

=0.00953

$$\Delta p_{ds} = \left(\frac{dp}{dL}\right)\Delta L = 0.00953\times 12440 = 118.53 \text{ psi}$$

- Annular Flow

a. Velocity:

$$v_a = \frac{0.408q}{(D_2{}^2 - D_1^2)} = \frac{0.408 \times 100}{(10.711^2 - 5^2)} = 0.4547 \text{ ft/sec}$$

b. Reynolds number:

$$N_{\mathrm{Re}} = \frac{109000 v_a{}^{2-n} \rho}{k} \left( \frac{0.0208(D_2 - D_1)}{2 + \frac{1}{n}} \right)^n$$

$$N_{\mathrm{Re}} = \frac{109000 \times 0.4547^{2-0.6652} \times 11.55}{467.058} \left( \frac{0.0208 \times (10.711 - 5)}{2 + \frac{1}{0.6652}} \right)^{0.6652} = 99.111$$

c. For laminar flow, critical value

$N_{\mathrm{Re}c} = 3470\text{-}1370n = 3470\text{-}1370 \times 0.6652 = 2558.7$

For turbulent flow, critical value

$N_{\mathrm{Re}c} = 4270\text{-}1370n = 4270\text{-}1370 \times 0.6652 = 3358.7$

d. Regime flow determination:

Comparison between $N_{\mathrm{Re}}$ and $N_{\mathrm{Re}c}$

If $N_{\mathrm{Re}} < N_{Rec} \rightarrow$flow is laminar.

Friction factor is included in Eq. 5.23.

e. Frictional pressure loss calculation inside annulus:

*Laminar:*

$$\left(\frac{dp}{dL}\right)=\frac{kv_a^n\left(\frac{2+1/n}{0.0208}\right)^n}{144000(D_2-D_1)^{1+n}}=\frac{467.058\times 0.4547^{0.6652}\times\left(\frac{2+1/0.6652}{0.0208}\right)^{0.6652}}{144000\times(10.711-5)^{1+0.6652}}$$

=0.00319psi/ft

$$\Delta p_a=\left(\frac{dp}{dL}\right)\Delta L=0.00319\times 12440 \text{ =39.727 psi}$$

Finally, the pump pressure:

$$\Delta p_{p=}\Delta p_s+\Delta p_{ds}+\Delta p_b+\Delta p_a=0+118.53+3.257+39.727=161.51 \text{ psi}$$

**$q_2$=665 gallon/min**

- Pipe Flow

a. Velocity:

$$v_p=\frac{0.408q}{D_p{}^2}=\frac{0.408\times 665}{(4.5)^2}=13.399 \text{ ft/sec}$$

b. Reynolds number:

$$N_{\mathrm{Re}}=\frac{89100v_p{}^{2-n}\rho}{k}\left(\frac{0.0416D_p}{3+\frac{1}{n}}\right)^n$$

$$N_{\mathrm{Re}}=\frac{89100\times 13.399^{2-0.6652}\times 11.55}{467.058}\left(\frac{0.0416\times 4.5}{3+\frac{1}{0.6652}}\right)^{0.6652}=8486.21$$

c. For laminar flow, critical value $N_{\mathrm{Re}c}$ = 2558.7

For turbulent flow, critical value $N_{\mathrm{Re}c}$ = 3358.7

d. Regime flow determination:

Comparison between $N_{Re}$ and $N_{Rec}$

If $N_{Re} > N_{Rec} \rightarrow$flow is turbulent.

$$a = \frac{\log n + 3.93}{50} = \frac{\log 0.6652 + 3.93}{50} = 0.075$$

$$b = \frac{1.75 - \log n}{7} = \frac{1.75 - \log 0.6652}{7} = 0.2753$$

$$f = \frac{a}{N_{\mathrm{Re}}{}^{b}} = \frac{0.075}{8486.21^{0.2753}} = 0.00622$$

e. Frictional pressure loss calculation inside drillstring:

*Turbulent:*

$$\left(\frac{dp}{dL}\right) = \frac{f v_p{}^2 \rho}{25.81 D_p} = \frac{0.00622 \times 13.399^2 \times 11.55}{25.81 \times 4.5} = 0.11097 \text{ psi/ft}$$

$$\Delta p_{ds} = \left(\frac{dp}{dL}\right)\Delta L = 0.11097 \times 12440 = 1380.49 \text{ psi}$$

- Annular Flow

a. Velocity:

$$v_a = \frac{0.408q}{(D_2{}^2 - D_1^2)} = \frac{0.408 \times 665}{(10.711^2 - 5^2)} = 3.024 \text{ ft/sec}$$

b. Reynolds number:

$$N_{\mathrm{Re}} = \frac{109000 \times 3.024^{2-0.6652} \times 11.55}{467.058}\left(\frac{0.0208 \times (10.711 - 5)}{2 + \frac{1}{0.6652}}\right)^{0.6652} = 1243.003$$

c. For laminar flow, critical value $N_{Rec} = 2558.7$

For turbulent flow, critical value $N_{Rec} = 3358.7$

d. Regime flow determination:

Comparison between $N_{Re}$ and $N_{Rec}$

If $N_{Re} < N_{Rec}$ →flow is laminar.

Friction factor is included in Eq. 5.23.

e. Frictional pressure loss calculation inside annulus:

*Laminar:*

$$\left(\frac{dp}{dL}\right) = \frac{kv_a^n \left(\frac{2+1/n}{0.0208}\right)^n}{144000(D_2 - D_1)^{1+n}}$$

$$= \frac{467.058 \times 3.027^{0.6652} \times \left(\frac{2+1/0.6652}{0.0208}\right)^{0.6652}}{144000 \times (10.711-5)^{1+0.6652}} = 0.01127 \text{ psi/ft}$$

$$\Delta p = \left(\frac{dp}{dL}\right) \times 12440 = 140.195 \text{ psi}$$

Finally, the pump pressure:

$\Delta p_{p=} \Delta p_s + \Delta p_{ds} + \Delta p_b + \Delta p_a$=0+1380.49+ 144.04+140.195= 1664.725 psi

**B-4 API FLOW**

**$q_1$=100 gallon/min**

- Pipe Flow

*a. Velocity:*

$$v_p = \frac{0.408q}{D_p^{\ 2}} = \frac{0.408 \times 100}{(4.5)^2} = 2.015 \text{ ft/sec}$$

b. Reynolds number:

$$n = 3.32 \log\left(\frac{R_{600}}{R_{300}}\right) = 3.32 \log\left(\frac{92}{58}\right) = 0.6652$$

$$k = \frac{5.10 R_{600}}{1022^{n}} = \frac{5.10 \times 92}{1022^{0.6651}} = 4.6718 \text{ dyne.sec}^{n}/\text{ft}^{2}$$

$$\mu_e = 100k\left(\frac{96 v_p}{D_p}\right)^{n-1}\left(\frac{3n+1}{4n}\right)^{n}$$

$$\mu_e = 100 \times 4.6718\left(\frac{96 \times 2.015}{4.5}\right)^{0.6652-1}\left(\frac{3 \times 0.6652 + 1}{4 \times 0.6652}\right)^{0.6652} = 143.510 \text{ cp}$$

$$N_{\text{Re}} = \frac{928 v_p \rho D_p}{\mu_e} = \frac{928 \times 2.015 \times 11.55 \times 4.5}{143.510} = 677.229$$

c. Critical value $N_{Rec} = 2100$.

d. Regime flow determination:

Comparison between $N_{Re}$ and $N_{Rec}$

If $N_{Re} < N_{Rec} \rightarrow$flow is laminar.

$f = 16/ N_{Re} = 16/ 677.229 = 0.02363$

e. Frictional pressure loss calculation inside drillstring:

$$\left(\frac{dp}{dL}\right) = \frac{f v_p^{\,2} \rho}{25.81 D_p} = \frac{0.02363 \times 2.015^2 \times 11.55}{25.81 \times 4.5} = 0.00954 \text{ psi/ft}$$

$$\Delta p_{ds} = \left(\frac{dp}{dL}\right)\Delta L = 0.00954 \times 12440 = 118.669 \, psi$$

- *Annular Flow*

a. Velocity:

$$v_a = \frac{0.408 q}{(D_2^{\,2} - D_1^2)} = \frac{0.408 \times 100}{(10.711^2 - 5^2)} = 0.4547 \text{ ft/sec}$$

b. Reynolds number:

$$n = 0.657 \log\left(\frac{R_{100}}{R_3}\right) = 0.657 \log\left(\frac{32}{8}\right) = 0.3955$$

$$k = \frac{5.10 R_{100}}{170.2^{n}} = \frac{5.10 \times 32}{170.2^{0.3955}} = 21.43 \text{ dyne.sec}^{n}/\text{ft}^2$$

$$\mu_e = 100k\left(\frac{144v}{D_2 - D_1}\right)^{n-1}\left(\frac{2n+1}{3n}\right)^{n}$$

$$= 100 \times 21.43\left(\frac{144 \times 0.4547}{10.711 - 5}\right)^{0.3955-1}\left(\frac{2 \times 0.3955 + 1}{3 \times 0.3955}\right)^{0.3955} = 577.239 \text{ cp}$$

$$N_{\mathrm{Re}} = \frac{928 v_a \rho (D_2 - D_1)}{\mu_e} = \frac{928 \times 0.4547 \times 11.55 \times (10.711 - 5)}{577.239} = 48.205$$

c. Critical value $N_{Rec} = 2100$.

d. Regime flow determination:

Comparison between $N_{Re}$ and $N_{Rec}$

If $N_{Re} < N_{Rec}$ →flow is laminar.

$f_a$= 24/ 48.205 =0.49788

e. Frictional pressure loss calculation inside annulus:

$$\left(\frac{dp}{dL}\right) = \frac{f v_a^{\,2} \rho}{25.81 (D_2 - D_1)} = \frac{0.49788 \times 0.4547^2 \times 11.55}{25.81 \times (10.711 - 5)} = 0.008066 \text{ psi/ft}$$

$$\Delta p_a = \left(\frac{dp}{dL}\right)\Delta L = 0.008066 \times 12440 = 100.339 \quad psi$$

Finally, the pump pressure:

$\Delta p_{p=} \Delta p_s + \Delta p_{ds} + \Delta p_b + \Delta p_a$=0+118.669+ 3.257+100.339= 222.25 psi

**$q_2$=665 gallon/min**

- Pipe Flow

a. Velocity:

$$v_p = \frac{0.408q}{D_p{}^2} = \frac{0.408\times 665}{(4.5)^2} = 13.399 \text{ ft/sec}$$

b. Reynolds number:

$$n = 3.32\log\left(\frac{R_{600}}{R_{300}}\right) = 3.32\log\left(\frac{92}{58}\right) = 0.6651$$

$$k = \frac{5.10R_{600}}{1022^n} = \frac{5.10\times 92}{1022^{0.6651}} = 4.6718 \text{ dyne.sec}^n/\text{ft}^2$$

$$\mu_e = 100k\left(\frac{96v_p}{D_p}\right)^{n-1}\left(\frac{3n+1}{4n}\right)^n$$

$$\mu_e = 100\times 4.6718\left(\frac{96\times 13.399}{4.5}\right)^{0.6651-1}\left(\frac{3\times 0.6651+1}{4\times 0.6651}\right)^{0.6651} = 76.063 \text{ cp}$$

$$N_{\text{Re}} = \frac{928v_p\rho D_p}{\mu_e} = \frac{928\times 13.399\times 11.55\times 4.5}{76.063} = 8496.512$$

c. Critical value $N_{\text{Re}c} = 2100$.

d. Regime flow determination:

Comparison between $N_{\text{Re}}$ and $N_{\text{Re}c}$

If $N_{\text{Re}} > N_{\text{Re}c}$ →flow is turbulent.

$$a = \frac{\log 0.6651 + 3.93}{50} = 0.075$$

$$b = \frac{1.75 - \log 0.6651}{7} = 0.2753$$

$$f = \frac{a}{N_{\text{Re}}{}^b} = \frac{0.075}{8496.512^{0.2753}} = 0.00621$$

e. Frictional pressure loss calculation inside drillstring:

$$\left(\frac{dp}{dL}\right)=\frac{fv_p{}^2\rho}{25.81D_p}=\frac{0.00621\times13.399^2\times11.55}{25.81\times4.5}=0.11093 \text{ psi/ft}$$

$$\Delta p_{ds}=\left(\frac{dp}{dL}\right)\Delta L=0.11093\times12440=1380.028 \quad psi$$

- Annular Flow

a. Velocity:

$$v_a=\frac{0.408q}{(D_2{}^2-D_1^2)}=\frac{0.408\times665}{(10.711^2-5^2)}=3.024 \text{ ft/sec}$$

b. Reynolds number:

$$n=0.657\log\left(\frac{R_{100}}{R_3}\right)=0.657\log\left(\frac{32}{8}\right)=0.3955$$

$$k=\frac{5.10R_{100}}{170.2^n}=\frac{5.10*32}{170.2^{0.3955}}=21.43 \text{ dyne.sec}^n/\text{ft}^2$$

$$\mu_e=100k\left(\frac{144v}{D_2-D_1}\right)^{n-1}\left(\frac{2n+1}{3n}\right)^n$$

$$=100k\left(\frac{144\times3.024}{10.711-5}\right)^{0.3955-1}\left(\frac{2\times0.3955+1}{3\times0.3955}\right)^{0.3955}=183.7093 \text{cp}$$

$$N_{\text{Re}}=\frac{928v_a\rho(D_2-D_1)}{\mu_e}=\frac{928\times3.024\times11.55\times(10.711-5)}{183.7093}=1007.57$$

c. Critical value $N_{\text{Rec}}$ = 2100.

d. Regime flow determination:

Comparison between $N_{\text{Re}}$ and $N_{\text{Rec}}$

$N_{\text{Re}} < N_{\text{Rec}} \rightarrow$flow is laminar.

$f$= 24/ $N_{\text{Re}}$ = 24/ 1007.57=0.02382

e. Frictional pressure loss calculation inside annulus:

$$\left(\frac{dp}{dL}\right)=\frac{fv_a{}^2\rho}{25.81(D_2-D_1)}=\frac{0.02382\times 0.4547^2\times 11.55}{25.81\times(10.711-5)}=0.017 psi/ft$$

$$\Delta p_a=\left(\frac{dp}{dL}\right)\Delta L=0.017\times 12440=212.28 psi$$

Finally, the pump pressure:

$$\Delta p_{p=}\Delta p_s+\Delta p_{ds}+\Delta p_b+\Delta p_a=0+1380.028+144.04+212.28=1736.35 \text{ psi}$$

## B-5 HERSCHEL-BULKLEY FLOW

### $q_1$=100 gallon/min

- Pipe Flow

a. Velocity:

$$v_p=\frac{0.408q}{D_p{}^2}=\frac{0.408\times 100}{(4.5)^2}=2.015 \text{ ft/sec}$$

b. Reynolds number:

$$C_c=1-\left(\frac{1}{2n+1}\right)\frac{\tau_0}{\tau_0+k\left[\frac{(3n+1)q}{n\pi(D_p/2)^3}\right]^n}$$

Where:

$D_p$ = 4.5/12=0.375 ft

$q$=100x0.002228=0.2228 $ft^3$/sec

$n$=0.7129, from Herschel-Bulkley model, Chapter IV

$k$ =0.6686 lbf.$sec^n$/100$ft^2$, from Herschel-Bulkley model, Chapter IV

$\tau_0$ =6.6582 lb/100 $ft^2$, from Herschel-Bulkley Model, Chapter IV

$$C_c = 1 - \left(\frac{1}{2\times 0.7129+1}\right)\frac{6.6582}{6.6582 + 0.6686\left[\frac{(3\times 0.7129+1)\times 0.2228}{0.7129\times\pi(0.375/2)^3}\right]^{0.7129}}$$

$C_c$=0.8396

$$N_{\mathrm{Re}} = \frac{2(3n+1)}{n}\left[\frac{\rho v_p{}^{(2-n)}\left(\frac{D_p}{2}\right)^n}{\tau_0\left(\frac{D_p}{2v_p}\right)^n + k\left(\frac{3n+1}{nC_c}\right)^n}\right]$$

Where:

$\rho$ = 11.55x7.48=86.394 lbm/ft$^3$

$$N_{\mathrm{Re}} = \frac{2(3\times 0.7129+1)}{0.7129}\left[\frac{86.394\times 2.015^{(2-0.7129)}\times\left(\frac{0.375}{2}\right)^{0.7129}}{6.6582\left(\frac{0.375}{2\times 2.015}\right)^{0.7129} + 0.6686\left(\frac{3\times 0.7129+1}{0.7129\times 0.8396}\right)^{0.7129}}\right]$$

$N_{\mathrm{Re}}$=166.96487

c. Critical value $N_{\mathrm{Re}c}$

$$y = \frac{\log(n)+3.93}{50} = \frac{\log(0.7129)+3.93}{50} = 0.07566$$

$$z = \frac{1.75-\log(n)}{7} = \frac{1.75-\log(0.7129)}{7} = 0.27099$$

$$N_{\mathrm{Re}c} = \left[\frac{4(3n+1)}{ny}\right]^{\frac{1}{1-z}} = \left[\frac{4(3\times 0.7129+1)}{0.7129\times 0.07566}\right]^{\frac{1}{1-0.27099}} = 1765.031$$

d. Regime flow determination:

Comparison between $N_{Re}$ and $N_{\mathrm{Re}c}$

If $N_{\mathrm{Re}} < N_{\mathrm{Re}c}$ →flow is laminar.

Friction factor is included in Eq. 5.34

*e. - Frictional pressure loss calculation inside drillstring:*

*Laminar:*

$$\left(\frac{dp}{dL}\right)=\frac{4k}{14400D_p}\left\{\left(\frac{\tau_0}{k}\right)+\left[\left(\frac{3n+1}{nC_c}\right)\left(\frac{8q}{\pi D_p{}^3}\right)\right]^n\right\}$$

$$\left(\frac{dp}{dL}\right)=\frac{4\times0.6686}{14400\times0.375}\left\{\left(\frac{6.6582}{0.6686}\right)+\left[\left(\frac{3\times0.7129+1}{0.7129\times0.8396}\right)\left(\frac{8\times0.2228}{\pi\times0.375^3}\right)\right]^{0.7129}\right\}$$

=0.01371 psi/ft

$$\Delta p_{ds}=\left(\frac{dp}{dL}\right)\Delta L=0.01371\times12440=170.556 \text{ psi}$$

- Annular Flow

a. Velocity:

$$v_a=\frac{0.408q}{(D_2{}^2-D_1^2)}=\frac{0.408\times100}{(10.711^2-5^2)}=0.4547 \text{ ft/sec}$$

b. Reynolds number:

$$C_a^*=1-\left(\frac{1}{n+1}\right)\frac{\tau_0}{\tau_0+k\left\{\left[\frac{2(2n+1)}{n((D_2/2)-(D_1/2))}\right]\left[\frac{q}{\pi((D_2/2)^2-(D_1/2)^2)}\right]\right\}^n}$$

$$C_a^*=1-\left(\frac{1}{0.7129+1}\right)\times$$

$$\frac{6.6582}{6.6582+0.6686\left\{\left[\frac{2(2\times0.7129+1)}{0.7129\left[((10.711/(2\times12))-(5/(2\times12))\right]}\right]\left[\frac{0.2228}{\pi\left[((10.711/(2\times12))^2-(5/(2\times12))^2\right]}\right]\right\}^{0.7129}}$$

$C^*_a$=0.64091

$$N_{\mathrm{Re}} = \frac{4(2n+1)}{n}\left\{\frac{\rho v_a^{\,2-n}\left(\frac{D_2 - D_1}{2}\right)^n}{\tau_0\left(\frac{D_2 - D_1}{2v_a}\right)^n + k\left[\frac{2(2n+1)}{nC_a^*}\right]^n}\right\}$$

$$N_{\mathrm{Re}} = \frac{4(2\times 0.7129+1)}{0.7129}\times$$

$$\left[\frac{86.394\times 0.4547^{(2-0.71229)}\times\left(\frac{(10.711/12)-(5/12)}{2}\right)^{0.7129}}{6.6582\left(\frac{(10.711/12)-(5/12)}{2\times 0.4547}\right)^{0.7129} + 0.6686\left(\frac{2(2\times 0.7129+1)}{0.7129\times 0.64091}\right)^{0.7129}}\right]$$

$N_{\mathrm{Re}}$=19.64

c. Critical value $N_{\mathrm{Re}c}$

$$y = \frac{\log(n)+3.93}{50} = \frac{\log(0.7129)+3.93}{50} = 0.07566$$

$$z = \frac{1.75-\log(n)}{7} = \frac{1.75-\log(0.7129)}{7} = 0.27099$$

$$N_{\mathrm{Re}c} = \left[\frac{8(2n+1)}{ny}\right]^{\frac{1}{1-z}} = \left[\frac{8(2\times 0.7129+1)}{0.7129\times 0.07566}\right]^{\frac{1}{1-0.27099}} = 3207.697$$

d. Regime flow determination:

Comparison between $N_{\mathrm{Re}}$ and $N_{\mathrm{Re}c}$

If $N_{\mathrm{Re}} < N_{\mathrm{Re}c}$ →flow is laminar.

Friction factor is included in Eq. 5.40

e. Frictional pressure loss calculation inside annulus:

$$\left(\frac{dp}{dL}\right)=\frac{4k}{14400(D_2-D_1)}\left\{\left(\frac{\tau_0}{k}\right)+\left[\left(\frac{16(2n+1)}{nC_a^*(D_2-D_1)}\right)\left(\frac{q}{\pi(D_2^2-D_1^2)}\right)\right]^n\right\}$$

$$\left(\frac{dp}{dL}\right)=\frac{4\times0.6686}{14400(10.711/12-5/12)}\left\{\left(\frac{6.6582}{0.6686}\right)+\left[\left(\frac{16(2\times0.7129+1)}{0.7129\times0.64091(10.711/12-5/12)}\right)\left(\frac{0.2228}{\pi((10.711/12)^2-(5/12)^2)}\right)\right]^{0.7129}\right\}$$

*(dp/dL)*=0.007625 psi/ft

$$\Delta p_a=\left(\frac{dp}{dL}\right)\Delta L=0.007625\times12440=94.849\ psi$$

*Finally:*

$\Delta p_{p=}\Delta p_s+\Delta p_{ds}+\Delta p_b+\Delta p_a$=0+170.556+ 3.257+94.849= 268.661 psi

**$q_2$=665 gallon/min**

- Pipe Flow

a. Velocity:

$$v_p=\frac{0.408q}{D_p{}^2}=\frac{0.408\times665}{(4.5)^2}=13.399\ \text{ft/sec}$$

b. Reynolds number:

$$C_c=1-\left(\frac{1}{2n+1}\right)\frac{\tau_0}{\tau_0+k\left[\frac{(3n+1)q}{n\pi(D_p/2)^3}\right]^n}$$

Where:

$D_p$ = 4.5/12=0.375 ft

*q*=665x0.002228=1.48162 $ft^3$/sec

*n*=0.7129, from Herschel-Bulkley model, Chapter IV

*k* =0.6686 lbf.$sec^n$/100$ft^2$, from Herschel-Bulkley model, Chapter IV

$\tau_0$ =6.6582 lb/100 ft$^2$, from Herschel-Bulkley Model, Chapter IV

$$C_c = 1 - \left(\frac{1}{2\times 0.7129+1}\right)\frac{6.6582}{6.6582 + 0.6686\left[\frac{(3\times 0.7129+1)\times 1.48162}{0.7129\times\pi(0.375/2)^3}\right]^{0.7129}}$$

$C_c$=0.9417

$$N_{\mathrm{Re}} = \frac{2(3n+1)}{n}\left[\frac{\rho v_p{}^{(2-n)}\left(\frac{D_p}{2}\right)^n}{\tau_0\left(\frac{D_p}{2v_p}\right)^n + k\left(\frac{3n+1}{nC_c}\right)^n}\right]$$

Where:

$\rho$ = 11.55x7.48=86.394 lbm/ft$^3$

$$N_{\mathrm{Re}} = \frac{2(3\times 0.7129+1)}{0.7129}\left[\frac{86.394\times 13.399^{(2-0.7129)}\times\left(\frac{0.375}{2}\right)^{0.7129}}{6.6582\left(\frac{0.375}{2\times 13.399}\right)^{0.7129} + 0.6686\left(\frac{3\times 0.7129+1}{0.7129\times 0.9417}\right)^{0.7129}}\right]$$

$N_{\mathrm{Re}}$=2800.24

c. Critical value $N_{\mathrm{Re}c}$

$$y = \frac{\log(n)+3.93}{50} = \frac{\log(0.7129)+3.93}{50} = 0.07566$$

$$z = \frac{1.75-\log(n)}{7} = \frac{1.75-\log(0.7129)}{7} = 0.27099$$

$$N_{\mathrm{Re}c} = \left[\frac{4(3n+1)}{ny}\right]^{\frac{1}{1-z}} = \left[\frac{4(3\times 0.7129+1)}{0.7129\times 0.07566}\right]^{\frac{1}{1-0.27099}} = 1765.031$$

$N_{\mathrm{Re}c}$=1765.031

d. Regime flow determination:

Comparison between $N_{Re}$ and $N_{Rec}$

If $N_{Re} > N_{Rec}$ →flow is turbulent.

$$f = y(C_c N_{\mathrm{Re}})^{-z} = 0.07566(0.9417 \times 2800.24)^{-0.27099} = 0.00895$$

e. Frictional pressure loss calculation inside drillstring:

$$\left(\frac{dp}{dL}\right) = \frac{fq^2\rho}{144 \times \pi^2 \times D_p{}^5} = \frac{0.00895 \times 1.48162^2 \times 86.394}{1421.22(4.5/12)^5} \text{ 0.161033 psi/ft}$$

$$\Delta p_{ds} = \left(\frac{dp}{dL}\right)\Delta L = 0.161033 \times 12440 = 2003.254 \ \ psi$$

- Annular Flow

a. Velocity:

$$v_a = \frac{0.408q}{(D_2{}^2 - D_1^2)} = \frac{0.408 \times 665}{(10.711^2 - 5^2)} = 3.024 \text{ ft/sec}$$

b. Reynolds number:

$$C_a^* = 1 - \left(\frac{1}{n+1}\right) \frac{\tau_0}{\tau_0 + k\left\{\left[\frac{2(2n+1)}{n((D_2/2) - (D_1/2))}\right]\left[\frac{q}{\pi((D_2/2)^2 - (D_1/2)^2)}\right]\right\}^n}$$

$$C_a^* = 1 - \left(\frac{1}{0.7129+1}\right) \times$$

$$\frac{6.6582}{6.6582 + 0.6686\left\{\left[\frac{2(2 \times 0.7129 + 1)}{0.7129\left[((10.711/(2 \times 12)) - (5/(2 \times 12))\right]}\right]\left[\frac{1.48162}{\pi\left[((10.711/(2 \times 12))^2 - (5/(2 \times 12))^2\right]}\right]\right\}^{0.7129}}$$

$C^*_a$=0.8291

$$N_{\mathrm{Re}} = \frac{4(2n+1)}{n}\left\{\frac{\rho v_a^{\,2-n}\left(\frac{D_2 - D_1}{2}\right)^n}{\tau_0\left(\frac{D_2 - D_1}{2v_a}\right)^n + k\left[\frac{2(2n+1)}{nC_a^*}\right]^n}\right\}$$

$$N_{\mathrm{Re}} = \frac{4(2\times 0.7129+1)}{0.7129}\times$$

$$\left[\frac{86.394\times 3.024^{(2-0.7129)}\times\left(\frac{(10.711/12)-(5/12)}{2}\right)^{0.7129}}{6.6582\left(\frac{(10.711/12)-(5/12)}{2\times 3.024}\right)^{0.7129} + 0.6686\left(\frac{2(2\times 0.7129+1)}{0.7129\times 0.8291}\right)^{0.7129}}\right]$$

$N_{\mathrm{Re}}$= 429.6

c. Critical value $N_{\mathrm{Re}c}$

$$y = \frac{\log(n)+3.93}{50} = \frac{\log(0.7129)+3.93}{50} = 0.07566$$

$$z = \frac{1.75-\log(n)}{7} = \frac{1.75-\log(0.7129)}{7} = 0.27099$$

$$N_{\mathrm{Re}c} = \left[\frac{8(2n+1)}{ny}\right]^{\frac{1}{1-z}} = \left[\frac{8(2\times 0.7129+1)}{0.7129\times 0.07566}\right]^{\frac{1}{1-0.27099}} = 3207.697$$

d. Regime flow determination:

Comparison between $N_{\mathrm{Re}}$ and $N_{\mathrm{Re}c}$

If $N_{\mathrm{Re}} < N_{\mathrm{Re}c}$ →flow is laminar.

Friction factor is included in Eq. 5.40

e. Frictional pressure loss calculation inside annulus:

$$\left(\frac{dp}{dL}\right)=\frac{4k}{14400(D_2-D_1)}\left\{\left(\frac{\tau_0}{k}\right)+\left[\left(\frac{16(2n+1)}{n*C_a^*(D_2-D_1)}\right)\left(\frac{q}{\pi(D_2^2-D_1^2)}\right)\right]^n\right\}$$

$$\left(\frac{dp}{dL}\right)=\frac{4\times0.6686}{14400(10.711/12-5/12)}\left\{\left(\frac{6.6582}{0.6686}\right)+\left[\left(\frac{16(2\times0.7129+1)}{0.7129\times0.8291\times(10.711/12-5/12)}\right)\left(\frac{1.48162}{\pi((10.711/12)^2-(5/12)^2)}\right)\right]^{0.7129}\right\}$$

*(dp/dL)*=0.014614psi/ft

$$\Delta p_a=\left(\frac{dp}{dL}\right)\Delta L=0.014614\times12440=181.79\ psi$$

*Finally:*

$\Delta p_{p=}\Delta p_s+\Delta p_{ds}+\Delta p_b+\Delta p_a$=0+2003.254+ 144.04+181.79= 2329.084 psi

**B-6 UNIFIED FLOW**

***q*$_1$=100 gallon/min**

- Pipe Flow

a. Velocity:

$$v_p=\frac{24.5q}{D_p{}^2}=\frac{24.5\times100}{4.5^2}=121.988\ \text{ft/min}$$

b. Number of Reynolds:

$\mu_p = R_{600}- R_{300}$=92-58= 34 cp

$\tau_y = R_{300}-\mu_p$= 58-34= 24 lbf/100 ft$^2$

$$n_p=3.32\log\left(\frac{2\mu_p+\tau_y}{\mu_p+\tau_y}\right)=3.32\log\left(\frac{2\times34+24}{34+24}\right)=0.665.$$

$$k_p=1.066\left(\frac{\mu_p+\tau_y}{511^{n_p}}\right)=1.066\left(\frac{34+24}{511^{0.665}}\right)=0.971\ \text{lbf.sec}^n/100\text{ft}^2.$$

$\tau_{yL}$= (2$R_3$-$R_6$)1.066= (2x8-10) x1.066= 6.396 lbf/100ft$^2$

$$G = \frac{3n+1}{4n} = \frac{3\times 0.665+1}{4\times 0.665} = 1.126$$

$$\gamma_w = \frac{1.6Gv_p}{D_p} = \frac{1.6\times 1.126\times 121.988}{4.5} = 48.836 \text{ 1/sec}$$

$$\tau_w = \left(\frac{4}{3}\right)^n \tau_0 + k\gamma_w^n = \left(\frac{4}{3}\right)^{0.665} \times 6.394 + 0.971\times 48.836^{0.665} = 20.634 \text{ lbf/100 ft}^2$$

$$N_{\mathrm{Re}} = \frac{\rho v_p{}^2}{19.36\tau_w} = \frac{11.55\times 121.988^2}{19.36\times 20.634} = 430.26$$

c. Friction factor determination for any flow regime:

$$f_{\mathrm{laminar}} = 16 / N_{\mathrm{Re}} = 16 / 430.26 = 0.03719$$

$$f_{transient} = \frac{16N_{\mathrm{Re}}}{(3470-1370n)^2} = \frac{16\times 430.26}{(3470-1370\times 0.665)^2} = 0.00105$$

$$a = \frac{\log(0.665)+3.93}{50} = 0.076$$

$$b = \frac{1.75-\log(0.665)}{7} = 0.275$$

$$f_{turbulent} = \frac{a}{N_{\mathrm{Re}}{}^b} = \frac{0.076}{430.26^{0.275}} = 0.01433$$

$$f_{\mathrm{partial}} = \left(f_{\mathrm{transient}}^{-8} + f_{\mathrm{turbulent}}^{-8}\right)^{-1/8} = \left(0.00105^{-8} + 0.01433^{-8}\right)^{-1/8} = 0.00105$$

$$f = \left(f_{\mathrm{partial}}{}^{12} + f_{\mathrm{laminar}}{}^{12}\right)^{1/12} = \left(0.00105^{12} + 0.03719^{12}\right)^{1/12} = 0.03719$$

d. Frictional pressure loss calculation inside drillstring:

$$\left(\frac{dp}{dL}\right) = \frac{1.076 f v_p{}^2 \rho}{10^5 D_p} = \frac{1.076\times 0.03719\times 121.988^2\times 11.55}{10^5\times 4.5} = 0.01529 \text{ psi/ft}$$

$$\Delta p = \left(\frac{dp}{dL}\right)\Delta L == 0.01529\times 12440 = 190.14 \text{psi}$$

- Annular Flow

a. Velocity:

$$v_a = \frac{24.5q}{D_2{}^2 - D_1{}^2} = \frac{24.5\times100}{10.711^2 - 5^2} = 27.306 \text{ft/min}$$

b. Reynolds number:

$$n_a = 3.32\log\left(\frac{2\mu_p + \tau_y - \tau_o}{\mu_p + \tau_y - \tau_0}\right) = 3.32\log\left(\frac{2\times34 + 24 - 6.396}{34 + 24 - 6.396}\right) = 0.729$$

$$k_a = 1.066\left(\frac{\mu_p + \tau_y - \tau_0}{511^{n_a}}\right) = 1.066\left(\frac{34 + 24 - 6.396}{511^{0.73}}\right) = 0.577 \text{ lbf.sec}^n/100\text{ft}^2$$

$$G = \left(\frac{2n+1}{3n}\right)\times1.5 = \left(\frac{2\times0.729+1}{3\times0.729}\right)\times1.5 = 1.68$$

$$\gamma_w = \frac{1.6\times G\times v}{D_2 - D_1} = \frac{1.6\times1.68\times27.306}{10.711-5} = 12.89 \text{ 1/sec}$$

$$\tau_w = \left(\frac{3}{2}\right)^n \tau_0 + k\gamma_w^n = \left(\frac{3}{2}\right)^{0.729}\times6.396 + 0.577\times12.89^{0.729} = 12.33 \text{ Lbf/100 ft}^2$$

$$N_{\text{Re}} = \frac{\rho\times v^2}{19.36\times\tau_w} = \frac{11.55\times27.306^2}{19.36\times12.33} = 36.08$$

c. Friction factor determination for any flow regime:

$$f_{laminar} = 24/N_{\text{Re}} = 24/36.08 = 0.6652$$

$$f_{\text{transient}} = \frac{16\times N_{\text{Re}}}{(3470 - 1370n)^2} = \frac{16\times36.08}{(3470 - 1370\times0.729)^2} = 0.000095$$

$$a = \frac{\log(0.729) + 3.93}{50} = 0.076$$

$$b = \frac{1.75 - \log(0.729)}{7} = 0.269$$

$$f_{\text{turbulent}} = \frac{a}{N_{\text{Re}}{}^{b}} = \frac{0.076}{36.08^{0.269}} = 0.0289$$

$$f_{\text{partial}} = \left(f_{\text{transient}}^{-8} + f_{\text{turbulent}}^{-8}\right)^{-1/8} = \left(0.000095^{-8} + 0.0289^{-8}\right)^{-1/8} = 0.000095$$

$$f_a = \left(f_{\text{partial}}{}^{12} + f_{\text{laminar}}{}^{12}\right)^{1/12} = \left(0.000095^{12} + 0.6652^{12}\right)^{1/12} = 0.6652$$

d. Frictional pressure loss calculation inside annulus:

$$\left(\frac{dP}{dL}\right) = \frac{1.076 \times f_a \times v^2 \times \rho}{10^5 (D_2 - D_1)} = \frac{1.076 \times 0.6652 \times 27.306^2 \times 11.55}{10^5 (10.711 - 5)} = 0.0108 \text{ psi/ft}$$

$$\Delta p = \left(\frac{dp}{dL}\right)\Delta L = 0.0108 \times 12440 = 134.26 \text{ psi}$$

Finally:

$\Delta p_{p=} \Delta p_s + \Delta p_{ds} + \Delta p_b + \Delta p_a$=0+190.14+ 3.257+134.96= 327.66 psi

**$q_2$=665 gallon/min**

- Pipe Flow

a. Velocity:

$$v_p = \frac{24.5q}{D_p{}^2} = \frac{24.5 \times 665}{4.5^2} = 804.568 \text{ ft/min}$$

b. Number of Reynolds:

$\mu_p = R_{600} - R_{300}$=92-58= 34 cp

$\tau_y = R_{300} - \mu_p$ = 58-34= 24 lbf/100 ft$^2$

$$n_p = 3.32 \log\left(\frac{2\mu_p + \tau_y}{\mu_p + \tau_y}\right) = 3.32 \log\left(\frac{2 \times 34 + 24}{34 + 24}\right) = 0.665.$$

$$k_p = 1.066\left(\frac{\mu_p + \tau_y}{511^{n_p}}\right) = 1.066\left(\frac{34 + 24}{511^{0.665}}\right) = 0.971 \text{ lbf.sec}^n/100\text{ft}^2.$$

$\tau_{yL}$= (2$R_3$-$R_6$)1.066= (2x8-10) x1.066= 6.396 lbf/100ft$^2$

$$G = \frac{3n+1}{4n} = \frac{3\times 0.665+1}{4\times 0.665} = 1.126$$

$$\gamma_w = \frac{1.6Gv_p}{D_p} = \frac{1.6\times 1.126\times 804.568}{4.5} = 322.11 \text{ 1/sec}$$

$$\tau_w = \left(\frac{4}{3}\right)^n \tau_0 + k\gamma_w^n = \left(\frac{4}{3}\right)^{0.665} \times 6.394 + 0.971\times 322.11^{0.665} = 52.935 \text{ lbf/100 ft}^2$$

$$N_{\mathrm{Re}} = \frac{\rho v_p{}^2}{19.36\tau_w} = \frac{11.55\times 804.568^2}{19.36\times 52.935} = 7295.601$$

c. Friction factor determination for any flow regime:

$$f_{\mathrm{laminar}} = 16/N_{\mathrm{Re}} = 16/7295.601 = 0.00219$$

$$f_{transient} = \frac{16N_{\mathrm{Re}}}{(3470-1370n)^2} = \frac{16\times 7295.601}{(3470-1370\times 0.665)^2} = 0.01783$$

$$a = \frac{\log(0.665)+3.93}{50} = 0.076$$

$$b = \frac{1.75-\log(0.665)}{7} = 0.275$$

$$f_{turbulent} = \frac{a}{N_{\mathrm{Re}}{}^b} = \frac{0.076}{7295.601^{0.275}} = 0.006584$$

$$f_{\mathrm{partial}} = \left(f_{\mathrm{transient}}^{-8} + f_{\mathrm{turbulent}}^{-8}\right)^{-1/8} = \left(0.01783^{-8} + 0.006584^{-8}\right)^{-1/8} = 0.006584$$

$$f = \left(f_{\mathrm{partial}}{}^{12} + f_{\mathrm{laminar}}{}^{12}\right)^{1/12} = \left(0.006584^{12} + 0.00219^{12}\right)^{1/12} = 0.00658$$

d. Frictional pressure loss calculation inside drillstring:

$$\left(\frac{dp}{dL}\right) = \frac{1.076 f v_p{}^2 \rho}{10^5 D_p} = \frac{1.076\times 0.00658\times 804.568^2\times 11.55}{10^5\times 4.5} = 0.1176 \text{ psi/ft}$$

$$\Delta p = \left(\frac{dp}{dL}\right)\Delta L == 0.1176 \times 12440 = 1463.368 \text{psi}$$

- Annular Flow

a. Velocity:

$$v_a = \frac{24.5q}{D_2{}^2 - D_1{}^2} = \frac{24.5 \times 665}{10.711^2 - 5^2} = 181.582 \text{ ft/min}$$

b. Reynolds number:

$$n_a = 3.32 \log\left(\frac{2\mu_p + \tau_y - \tau_o}{\mu_p + \tau_y - \tau_0}\right) = 3.32 \log\left(\frac{2 \times 34 + 24 - 6.396}{34 + 24 - 6.396}\right) = 0.729$$

$$k_a = 1.066\left(\frac{\mu_p + \tau_y - \tau_0}{511^{n_a}}\right) = 1.066\left(\frac{34 + 24 - 6.396}{511^{0.729}}\right) = 0.577 \text{ lbf.sec}^n/100\text{ft}^2$$

$$G = \left(\frac{2n+1}{3n}\right) \times 1.5 = \left(\frac{2 \times 0.729 + 1}{3 \times 0.729}\right) \times 1.5 = 1.68$$

$$\gamma_w = \frac{1.6 \times G \times v}{D_2 - D_1} = \frac{1.6 \times 1.68 \times 181.582}{10.711 - 5} = 85.76 \text{ 1/sec}$$

$$\tau_w = \left(\frac{3}{2}\right)^n \tau_0 + k\gamma_w^n = \left(\frac{3}{2}\right)^{0.729} \times 6.396 + 0.577 \times 85.76^{0.729} = 23.47 \text{Lbf/100 ft}^2$$

$$N_{\text{Re}} = \frac{\rho \times v^2}{19.36 \times \tau_w} = \frac{11.55 \times 181.582^2}{19.36 \times 23.47} = 838.69$$

c. Friction factor determination for any flow regime:

$$f_{laminar} = 24 / N_{\text{Re}} = 24 / 838.69 = 0.02861$$

$$f_{\text{transient}} = \frac{16 \times N_{\text{Re}}}{(3470 - 1370n)^2} = \frac{16 \times 838.69}{(3470 - 1370 \times 0.729)^2} = 0.00219$$

$$a = \frac{\log(0.729) + 3.93}{50} = 0.076$$

$$b = \frac{1.75 - \log(0.729)}{7} = 0.269$$

$$f_{\text{turbulent}} = \frac{a}{N_{\text{Re}}{}^{b}} = \frac{0.076}{838.69^{0.269}} = 0.01223$$

$$f_{\text{partial}} = \left(f_{\text{transient}}^{-8} + f_{\text{turbulent}}^{-8}\right)^{-1/8} = \left(0.00219^{-8} + 0.01223^{-8}\right)^{-1/8} = 0.00219$$

$$f_a = \left(f_{\text{partial}}{}^{12} + f_{\text{laminar}}{}^{12}\right)^{1/12} = \left(0.00219^{12} + 0.02861^{12}\right)^{1/12} = 0.02861$$

d. Frictional pressure loss calculation inside annulus:

$$\left(\frac{dP}{dL}\right) = \frac{1.076 \times f_a \times v^2 \times \rho}{10^5 (D_2 - D_1)} = \frac{1.076 \times 0.02861 \times 181.58^2 \times 11.55}{10^5 (10.711 - 5)} = 0.02053 \text{ psi/ft}$$

$$\Delta p = \left(\frac{dp}{dL}\right)\Delta L = 0.02053 \times 12440 = 255.42 \text{ psi}$$

Finally:

$\Delta p_{p=} \Delta p_s + \Delta p_{ds} + \Delta p_b + \Delta p_a$=0+1463.368+ 144.04+255.42= 1862.82 psi

**B-7 ROBERTSON AND STIFF FLOW**

***q*$_1$=100 gallon/min**

- Pipe Flow

a. Velocity:

$$v_p = \frac{0.408q}{D_p{}^2} = \frac{0.408 \times 100}{(4.5)^2} = 2.015 \text{ ft/sec}$$

b. Reynolds number:

$$N_{\mathrm{Re}} = \frac{89100 v_p^{\,2-B} \rho}{A} \left( \frac{0.0416 D_p}{3 + \frac{1}{B}} \right)^B$$

$$= \frac{89100 \times 2.015^{2-0.6186} \times 11.55}{628.65} \left( \frac{0.0416 \times 4.5}{3 + \frac{1}{0.6186}} \right)^{0.6186}$$

=593.305

Where:

*A*=628.65 cp*,* from Robertson and Stiff, Chapter IV

*B*= 0.6186, from Robertson and Stiff, Chapter IV

*c*. For laminar flow, critical value

$N_{\mathrm{Re}c}$ = 3470-1370B= 3470-1370x0.6186=2622.55

For turbulent flow, critical value

$N_{\mathrm{Re}c}$ = 4270-1370B= 4270-1370x0.6186=3422.55

d. Regime flow determination:

Comparison between $N_{\mathrm{Re}}$ and $N_{\mathrm{Re}c}$

If $N_{\mathrm{Re}} < N_{\mathrm{Re}c} \rightarrow$flow is laminar.

Friction factor is included in Eq. 5.62

e. Frictional pressure loss calculation inside drillstring:

*Laminar:*

$$\left(\frac{dp}{dL}\right)=8.33E-4\times 2^{2+B}\times A\left\{\left(\frac{1+3B}{B}\right)\left[\frac{0.2v_p+\frac{C}{6}D_p}{D_p^{\left(\frac{1+B}{B}\right)}}\right]\right\}^B$$

$$\left(\frac{dp}{dL}\right)=8.33E-4\times 2^{2+0.6186}\times 1.313\left\{\left(\frac{1+3\times 0.6186}{0.6186}\right)\left[\frac{0.2\times 120.88+\frac{17.121}{6}4.5}{4.5^{\left(\frac{1+0.6186}{0.6186}\right)}}\right]\right\}^{0.6186}$$

=0.01416psi/ft

Where:

*A*=1.313 lbf*sec$^B$/100 ft$^2$, from Robertson and Stiff, Chapter IV

C=17.121 1/sec $^B$, from Robertson and Stiff, Chapter IV

$v_p$= 2.015x60=120.88 ft/min

$$\Delta p=\left(\frac{dp}{dL}\right)\Delta L=0.01416\times 12440=176.1504\ psi$$

- Annular Flow

a. Velocity:

$$v_a=\frac{0.408q}{(D_2{}^2-D_1^2)}=\frac{0.408\times 100}{(10.711^2-5^2)}=0.4547\ \text{ft/sec}$$

b. - Reynolds number:

$$N_{\text{Re}}=\frac{109000v_a{}^{2-B}\rho}{A}\left(\frac{0.0208(D_2-D_1)}{2+\frac{1}{B}}\right)^B$$

$$N_{\text{Re}}=\frac{109000\times 0.4547^{2-0.6186}\times 11.55}{628.65}\left[\frac{0.0208\times(10.711-5)}{2+\frac{1}{0.6186}}\right]^{0.6186}=81.49$$

c. For laminar flow, critical value $N_{Rec}$ = 2622.55

For turbulent flow, critical value $N_{Rec}$ = 3422.55

d. Regime flow determination:

Comparison between $N_{Re}$ and $N_{Rec}$

If $N_{Re} < N_{Rec} \rightarrow$flow is laminar.

Friction factor is included in Eq. 5.64.

e. Frictional pressure loss calculation inside annulus:

*Laminar:*

$$\left(\frac{dp}{dL}\right) = 8.33E-4\times 4^{1+B}\times A\left\{\left(\frac{1+2B}{B}\right)\left[\frac{0.2v_a+\frac{C}{8}(D_2-D_1)}{(D_2-D_1)^{\left(\frac{1+B}{B}\right)}}\right]\right\}^B$$

$$\left(\frac{dp}{dL}\right) = 8.33E-4\times 4^{1+0.6186}\times 1.313\times\left\{\left(\frac{1+2\times 0.6186}{0.6186}\right)\left[\frac{0.2\times 27.28+\frac{17.121}{8}(10.711-5)}{(10.711-5)^{\left(\frac{1+0.6186}{0.6186}\right)}}\right]\right\}^{0.6186}$$

=0.00805 psi/ft

Where

$V_a$= 0.4547x60=27.28 ft/min

$$\Delta p = \left(\frac{dp}{dL}\right)\Delta L = 0.00805\times 12440 = 100.142 \text{ psi}$$

Finally:

$\Delta p_{p=} \Delta p_s + \Delta p_{ds} + \Delta p_b + \Delta p_a$=0+176.1504+ 3.257+100.142= 279.549 psi

Note: To consider yield stress with this model, use the following equations and estimate the frictional pressure loss for laminar flow by iteration:

- Pipe flow

$$\lambda = \frac{2(AC)^B}{\left(\frac{dp}{dL}\right)} = \frac{2(0.0131\times 17.12)^{0.6186}}{\left(\frac{dp}{dL}\right)} = \frac{0.7933}{\left(\frac{dp}{dL}\right)}$$

- Annular Flow

$$\lambda = \frac{(AC)^B}{\left(\frac{dp}{dL}\right)} = \frac{0.3966}{\left(\frac{dp}{dL}\right)}$$

Where:

$A$ = 0.0131 Lbf.sec$^B$/ft$^2$

$C$= 17.12 1/sec$^B$

$dp/dL$= lbf/ft$^2$/ft

General equation to estimate friction pressure loss:

- Pipe flow

$$q = \pi\left\{\left[\frac{1}{2A}\left(\frac{dp}{dL}\right)\right]^{1/B}\left(\frac{B}{3B+1}\right)\left[\left(\frac{D}{2}\right)^{\frac{3B+1}{B}} - \lambda^{\frac{3B+1}{B}}\right] - \frac{C}{3}\left[\left(\frac{D}{2}\right)^3 - \lambda^3\right]\right\}$$

$$0.2228 = \pi\left\{\left[\frac{1}{2\times 0.0131}\left(\frac{dp}{dL}\right)\right]^{1/0.6186}\left(\frac{0.6186}{3\times 0.6186+1}\right)\left[\left(\frac{4.5}{2}\right)^{\frac{3\times 0.6186+1}{0.6186}} - \lambda^{\frac{3\times 0.6186+1}{0.6186}}\right] - \frac{17.12}{3}\left[\left(\frac{4.5}{2}\right)^3 - \lambda^3\right]\right\}$$

Applying Solve from Excel, we have dp/dL= 0.0304 psi/ft. Note that λ is also function of dp/dL.

- Annular Flow

$$q = 2(D_2 - D_1)\left\{\left[\frac{1}{2A}\left(\frac{dp}{dL}\right)\right]^{1/B}\left(\frac{B}{2B+1}\right)\left[\left(\frac{D}{2}\right)^{\frac{3B+1}{B}} - \lambda^{\frac{2B+1}{B}}\right] - \frac{C}{2}\left[\left(\frac{D}{2}\right)^3 - \lambda^3\right]\right\}$$

$$q = 2(10.711-5)\left\{\left[\frac{1}{2\times 0.0131}\left(\frac{dp}{dL}\right)\right]^{1/0.6186}\left(\frac{0.61.86}{2\times 0.6186+1}\right)\left[\left(\frac{10.711-5}{2}\right)^{\frac{3\times 0.6186+1}{0.6186}} - \lambda^{\frac{2\times 0.6186+1}{0.6186}}\right] - \frac{17.12}{2}\left[\left(\frac{10.711-5}{2}\right)^{3} - \lambda^{3}\right]\right\}$$

Applying Solve from Excel, we have dp/dL= 0.00806 psi/ft. Note that λ is also function of dp/dL.

Finally:

$\Delta p_{p=} \Delta p_s + \Delta p_{ds} + \Delta p_b + \Delta p_a$=0+0.0304x12440+ 3.257+0.00806x12440

= 481.277 psi

**$q_2$=665 gallon/min**

- Pipe Flow

a. Velocity:

$$v_p = \frac{0.408q}{D_p{}^2} = \frac{0.408\times 665}{(4.5)^2} = 13.399 \text{ ft/sec or } 803.94 \text{ ft/min}$$

b. Reynolds number:

$$N_{\mathrm{Re}} = \frac{89100 v_p{}^{2-B}\rho}{A}\left(\frac{0.0416 D_p}{3+\frac{1}{B}}\right)^{B} = \frac{89100\times 13.399^{2-0.6186}\times 11.55}{628.65}\left(\frac{0.0416\times 4.5}{3+\frac{1}{0.6186}}\right)^{0.6186}$$

=8126.25

Where:

*A*=628.65 cp*,* from Robertson and Stiff, Chapter IV

*B*= 0.6186, from Robertson and Stiff, Chapter IV

*c*. For laminar flow, critical value

$N_{Rec}$ = 3470-1370B= 3470-1370x0.6186=2622.55

For turbulent flow, critical value

$N_{Rec}$ = 4270-1370B= 4270-1370x0.6186=3422.55

d. Regime flow determination:

Comparison between $N_{Re}$ and $N_{Rec}$

If $N_{Re} > N_{Rec} \rightarrow$flow is Turbulent.

$$a = \frac{\log B + 3.93}{50} = \frac{\log 0.6186 + 3.93}{50} 0.0744$$

$$b = \frac{1.75 - \log B}{7} = \frac{1.75 - \log 0.6186}{7} 0.2798$$

$$f = \frac{a}{N_{\mathrm{Re}}{}^{b}} = \frac{0.0744}{8126.25^{0.2798}} = 0.00599$$

e. Frictional pressure loss calculation inside drillstring:

$$\left(\frac{dp}{dL}\right) = \frac{f v_p{}^2 \rho}{25.81 D_p} = \frac{0.00599 \times 13.399^2 \times 11.55}{25.81 \times 4.5} = 0.10698$$

$$\Delta p = \left(\frac{dp}{dL}\right) \Delta L = 0.010698 \times 12440 = 1330.86 \text{ psi}$$

- Annular Flow

a. Velocity:

$$v_a = \frac{0.408q}{(D_2{}^2 - D_1^2)} = \frac{0.408 \times 665}{(10.711^2 - 5^2)} = 3.024 \text{ ft/sec or } 181.44 \text{ ft/min}$$

b. - Reynolds number:

$$N_{\mathrm{Re}} = \frac{109000 v_a^{\,2-B} \rho}{A} \left( \frac{0.0208(D_2 - D_1)}{2 + \frac{1}{B}} \right)^B$$

$$N_{\mathrm{Re}} = \frac{109000 \times 3.024^{2-0.6186} \times 11.55}{628.65} \left[ \frac{0.0208 \times (10.711 - 5)}{2 + \frac{1}{0.6186}} \right]^{0.6186} = 1116.31$$

c. For laminar flow, critical value $N_{\mathrm{Re}c} = 2622.55$

For turbulent flow, critical value $N_{\mathrm{Re}c} = 3422.55$

d. Regime flow determination:

Comparison between $N_{\mathrm{Re}}$ and $N_{\mathrm{Re}c}$

If $N_{\mathrm{Re}} < N_{\mathrm{Re}c} \rightarrow$flow is laminar.

Friction factor is included in Eq. 5.64.

e. Frictional pressure loss calculation inside annulus:

*Laminar:*

$$\left( \frac{dp}{dL} \right) = 8.33E-4 \times 4^{1+B} \times A \left\{ \left( \frac{1+2B}{B} \right) \left[ \frac{0.2 v_a + \frac{C}{8}(D_2 - D_1)}{(D_2 - D_1)^{\left( \frac{1+B}{B} \right)}} \right] \right\}^B$$

$$\left( \frac{dp}{dL} \right) = 8.33E-4 \times 4^{1+0.6186} \times 1.313 \times \left\{ \left( \frac{1+2 \times 0.6186}{0.6186} \right) \left[ \frac{0.2 \times 181.44 + \frac{17.121}{8}(10.711 - 5)}{(10.711 - 5)^{\left( \frac{1+0.6186}{0.6186} \right)}} \right] \right\}^{0.6186}$$

=0.015023 psi/ft

$$\Delta p = \left( \frac{dp}{dL} \right) \Delta L = 0.00805 \times 12440 = 186.886 \text{ psi}$$

Finally:

$$\Delta p_{p=} \Delta p_s + \Delta p_{ds} + \Delta p_b + \Delta p_a = 0+1330.86 + 144.04+186.886 = 1661.78 \text{ psi}$$

**B-8 CASSON FLOW**

**$q_1$=100 gallon/min**

- Pipe Flow

a. Velocity:

$$v_p = \frac{0.408q}{D_p^{\ 2}} = \frac{0.408 \times 100}{(4.5)^2} = 2.015 \text{ ft/sec}$$

b. - Reynolds number:

$$N_{\text{Re}} = \frac{928 v_p \rho D_p}{\mu_c} = \frac{928 \times 2.015 \times 11.55 \times 4.5}{26.39} = 3682.8$$

Where:

$\mu_c$=26.39 cp, from Casson model, Chapter IV

*c.* Critical value $N_{\text{Re}c}$ from Fig.B1.

$$C_a = \frac{D_p^2 \tau_c \rho}{32.174 \mu_c^2} = \frac{(4.5/12)^2 \times 0.06524 \times 86.394}{32.174 \times 0.000551^2} = 81143$$

Where:

$\tau_c$=0.06524lbf/ft$^2$ from Casson model, Chapter IV

$\rho$=86.394 lbm/ft$^3$

$\mu_c$= 0.0005510 lbf.sec/ft$^2$

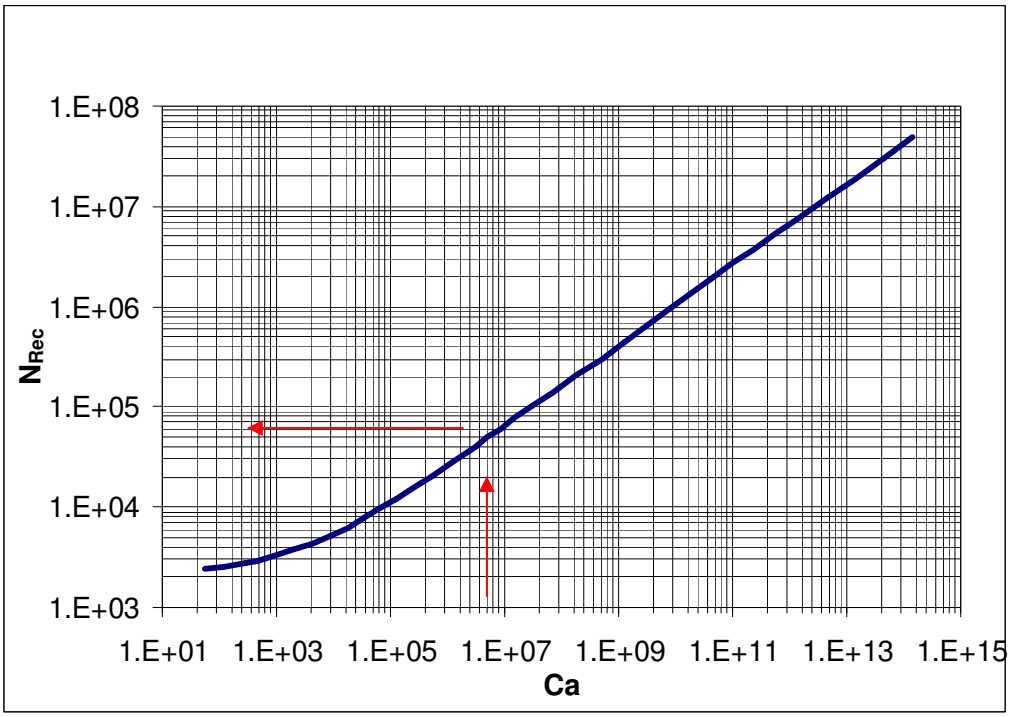



Fig. B1— Critical Reynolds numbers for Casson fluids-example followed.

d. Regime flow determination:

Comparison between $N_{Re}$ and $N_{Rec}$

$N_{Rec}$=14580 from Fig.B1.

If $N_{Re} < N_{Rec}$ →flow is laminar.

Friction factor is included in Eq. 5.70.

e. Frictional pressure loss calculation inside drillstring:

*Laminar:*

$$q = \frac{\pi D_p^{\,3}}{8\mu_c}\left[\frac{D_p\left(\frac{dp}{dL}\right)}{16} - \frac{4}{7}\sqrt{\tau_c}\sqrt{\frac{\left(\frac{dp}{dL}\right)D_p}{4}} - \frac{64\tau_c^{\,4}}{84 D_p^{\,3}\left(\frac{dp}{dL}\right)^3} + \frac{\tau_c}{3}\right]$$

Where:

$q$=0.2228 ft$^3$/sec

$D_p$=0.375 ft

$\mu_c$= 0.0005510 lbf.sec/ft$^2$

$\tau_c$=0.065 lbf/ft$^2$

dp/dL= lb/ft$^2$/ft

$$0.2228 = \frac{\pi \times 0.375^3}{8 \times 0.000551}\left[\frac{0.375\left(\frac{dp}{dL}\right)}{16} - \frac{4}{7}\sqrt{0.065}\sqrt{\frac{\left(\frac{dp}{dL}\right)0.375}{4}} - \frac{64 \times 0.065^4}{84 \times 0.375^3\left(\frac{dp}{dL}\right)^3} + \frac{0.065}{3}\right]$$

Applying Solve from Excel, we have dp/dL= 0.01436 psi/ft.

$$\Delta p = \left(\frac{dp}{dL}\right)\Delta L = 0.01436 \times 12440 = 178.6384 \text{ psi}$$

- Annular Flow

a. Velocity:

$$v_a = \frac{0.408q}{(D_2{}^2 - D_1^2)} = \frac{0.408 \times 100}{(10.711^2 - 5^2)} = 0.4547 \text{ ft/sec}$$

b. Reynolds number:

$$N_{\text{Re}} = \frac{757(D_2 - D_1)v_a\rho}{\mu_c} = \frac{757(10.711 - 5) \times 0.4547 \times 11.55}{26.39} = 860.35$$

c. Critical value $N_{\text{Rec}}$ from Fig. A1.

$$C_a = \frac{(D_2 - D_1)^2 \tau_c \rho}{32.174\mu_c^2} = \frac{\left(\frac{10.711}{12} - \frac{5}{12}\right)^2 \times 0.065 \times 86.394}{32.174 \times 0.00055^2} = 130685.779$$

d. Regime flow determination:

Comparison between $N_{\text{Re}}$ and $N_{\text{Rec}}$

$N_{\text{Rec}}$=17338 from Fig.B1.

If $N_{\text{Re}} < N_{\text{Rec}}$ →flow is laminar.

Friction factor is included in Eq. 5.72

e. Frictional pressure loss calculation inside annulus:

*Laminar:*

$$q = \frac{\pi\left(D_2 - D_1\right)^3}{8\mu_c} \times \left[ \frac{\left(D_2 - D_1\right)\left(\frac{dp}{dL}\right)}{16} - \frac{4}{7}\sqrt{\tau_c}\sqrt{\frac{\left(\frac{dp}{dL}\right)\left(D_2 - D_1\right)}{4}} - \frac{64\tau_c^{\,4}}{84\left(D_2 - D_1\right)^3\left(\frac{dp}{dL}\right)^3} + \frac{\tau_c}{3} \right]$$

Where

dp/dL= lb/ft$^2$/ft

$$0.2228 = \frac{\pi\left(\frac{10.711}{12} - \frac{5}{12}\right)^3}{8 \times 0.00055} \times$$

$$\left[ \frac{\left(\frac{10.711}{12} - \frac{5}{12}\right)\left(\frac{dp}{dL}\right)}{16} - \frac{4}{7}\sqrt{0.065}\sqrt{\frac{\left(\frac{dp}{dL}\right)\left(\frac{10.711}{12} - \frac{5}{12}\right)}{4}} - \frac{64 \times 0.065^{\,4}}{84\left(\frac{10.711}{12} - \frac{5}{12}\right)^3\left(\frac{dp}{dL}\right)^3} + \frac{0.065}{3} \right]$$

Applying Solve from Excel, we have dp/dL= 0.0090 psi/ft.

$$\Delta p = \left(\frac{dp}{dL}\right)\Delta L = 0.00898 \times 12440 = 111.69 \text{ psi}$$

Finally:

$$\Delta p_{p=} \Delta p_s + \Delta p_{ds} + \Delta p_b + \Delta p_a = 0 + 178.63 + 3.257 + 111.69 = 293.847 \text{ psi}$$

**$q_2$=665 gallon/min**

- Pipe Flow

a. Velocity:

$$v_p = \frac{0.408q}{D_p^{\,2}} = \frac{0.408 \times 665}{(4.5)^2} = 13.399 \text{ ft/sec}$$

b. - Reynolds number:

$$N_{\mathrm{Re}} = \frac{928 v_p \rho D_p}{\mu_c} = \frac{928 \times 13.399 \times 11.55 \times 4.5}{26.39} = 24489.25$$

Where:

$\mu_c$=26.39 cp, from Casson model, Chapter IV

*c.* Critical value $N_{Rec}$ from Fig.B1.

$$C_a = \frac{D_p^2 \tau_c \rho}{32.174 \mu_c^2} = \frac{(4.5/12)^2 \times 0.06524 \times 86.394}{32.174 \times 0.000551^2} = 81143.14$$

Where:

$\tau_c$=0.06524lbf/ft$^2$ from Casson model, Chapter IV

$\rho$=86.394 lbm/ft$^3$

$\mu_c$= 0.0005510 lbf.sec/ft$^2$

d. Regime flow determination:

Comparison between $N_{Re}$ and $N_{Rec}$

$N_{Rec}$=14577 from Fig.B1.

If $N_{Re} < N_{Rec} \rightarrow$flow is turbulent.

$$f = \frac{0.0791}{N_{\mathrm{Re}}^{0.25}} = \frac{0.0791}{24489.25^{0.25}} = 0.00632$$

e. Frictional pressure loss calculation inside drillstring:

$$\left(\frac{dp}{dL}\right) = \frac{f v_p^{\ 2} \rho}{25.81 D_p} = \frac{0.00632 \times 13.399^2 \times 11.55}{25.81 \times 4.5} = 0.11289$$

$$\Delta p = \left(\frac{dp}{dL}\right) \Delta L = 0.11289 \times 12440 = 1404.36 \text{ psi}$$

- Annular Flow

a. Velocity:

$$v_a = \frac{0.408q}{(D_2{}^2 - D_1^2)} = \frac{0.408 \times 665}{(10.711^2 - 5^2)} = 3.024 \text{ ft/sec}$$

b. Reynolds number:

$$N_{\text{Re}} = \frac{757(D_2 - D_1)v_a \rho}{\mu_c} = \frac{757(10.711 - 5) \times 3.024 \times 11.55}{26.39} = 5721.797$$

c. Critical value $N_{\text{Re}c}$ from Fig. A1.

$$C_a = \frac{(D_2 - D_1)^2 \tau_c \rho}{32.174 \mu_c^2} = \frac{\left(\frac{10.711}{12} - \frac{5}{12}\right)^2 \times 0.065 \times 86.394}{32.174 \times 0.00055^2} = 130685.779$$

d. Regime flow determination:

Comparison between $N_{\text{Re}}$ and $N_{\text{Re}c}$

$N_{\text{Re}c}$=17338 from Fig.B1.

If $N_{\text{Re}} < N_{\text{Re}c}$ →flow is laminar.

Friction factor is included in Eq. 5.72

e. Frictional pressure loss calculation inside annulus:

*Laminar:*

$$q = \frac{\pi(D_2 - D_1)^3}{8\mu_c} \times \left[ \frac{(D_2 - D_1)\left(\frac{dp}{dL}\right)}{16} - \frac{4}{7}\sqrt{\tau_c}\sqrt{\frac{\left(\frac{dp}{dL}\right)(D_2 - D_1)}{4}} - \frac{64\tau_c{}^4}{84(D_2 - D_1)^3\left(\frac{dp}{dL}\right)^3} + \frac{\tau_c}{3} \right]$$

Where:

$q$=1.48162 ft$^3$/sec

$\mu_c$= 0.0005510 lbf.sec/ft$^2$

$\tau_c$=0.065 lbf/ft$^2$

dp/dL= lb/ft$^2$/ft

$$1.48162 = \frac{\pi\left(\frac{10.711}{12}-\frac{5}{12}\right)^3}{8\times 0.00055}\times$$

$$\left[\frac{\left(\frac{10.711}{12}-\frac{5}{12}\right)\left(\frac{dp}{dL}\right)}{16}-\frac{4}{7}\sqrt{0.065}\sqrt{\frac{\left(\frac{dp}{dL}\right)\left(\frac{10.711}{12}-\frac{5}{12}\right)}{4}}-\frac{64\times 0.065^4}{84\left(\frac{10.711}{12}-\frac{5}{12}\right)^3\left(\frac{dp}{dL}\right)^3}+\frac{0.065}{3}\right]$$

Applying Solve from Excel, we have dp/dL= 0.01873psi/ft.

$$\Delta p = \left(\frac{dp}{dL}\right)\Delta L = 0.01873\times 12440 = 233.0012 \text{ psi}$$

Finally:

$\Delta p_{p=} \Delta p_s + \Delta p_{ds} + \Delta p_b + \Delta p_a$=0+1404.36 + 144.04+233.012 = 1781.415 psi

**Table B1** and **B2** show a comparison between pressure drop results calculated with the various models at two flow rates.

**Table B1— Pressure Drop Results Calculated with the Various Models (q=100 gpm)**

| Model | Newt. | B-P | Power Law | API-RP13D | H-B | Unified | R&S "original" | Casson |
|---|---|---|---|---|---|---|---|---|
| $\Delta p_{ds}$,psi | 47.846 | 249.7 | 118.53 | 118.66 | 170.56 | 190.14 | 176.15 | 178.73 |
| $\Delta p_a$,psi | 8.24 | 218.88 | 39.73 | 100.33 | 94.85 | 134.27 | 100.14 | 111.69 |
| $\Delta p_b$,psi | 3.26 | 3.26 | 3.26 | 3.26 | 3.26 | 3.26 | 3.26 | 3.26 |
| $\Delta p_p$,psi | 59.346 | 471.84 | 161.52 | 222.25 | 268.67 | 327.67 | 279.55 | 293.68 |

**Table B2— Pressure Drop Results Calculated with the Various Models (q=665 gpm)**

| Model | Newt. | B-P | Power Law | API-RP13D | H-B | Unified | R&S "original" | Casson |
|---|---|---|---|---|---|---|---|---|
| $\Delta p_{ds}$,psi | 1709.9 | 1819.15 | 1380.49 | 1380.03 | 2003.3 | 1463.4 | 1330.86 | 1404.4 |
| $\Delta p_a$,psi | 98.7 | 246.2 | 140.2 | 212.28 | 181.79 | 255.42 | 186.889 | 233.01 |
| $\Delta p_b$,psi | 144.04 | 144.04 | 144.04 | 144.04 | 144.04 | 144.04 | 144.04 | 144.04 |
| $\Delta p_p$,psi | 1952.7 | 2209.39 | 1664.73 | 1736.35 | 2329.13 | 1862.9 | 1661.789 | 1781.4 |

# APPENDIX C

## C-1 Enlargement and Contraction

Appendix C proposes a method to calculate hydraulics correcting by tool joints effect, for a flow rate of 100 gallon/min. Newtonian fluid.

- Pipe

**Gradual Enlargement**

**Table C1— Angles for Internal Upset (Drillpipe) for Given Example**

| Pipe | | | | tool joint | | | angle calculation | |
|---|---|---|---|---|---|---|---|---|
| **OD,in** | **wt(lbf/ft)** | **Grade** | **d,in** | **miu,in** | **dou,in** | **d-dou** | **θ/2** | **θ** |
| 4.5 | 20 | D,E | 3.6400 | 2 | 3.0000 | 0.6400 | 17.7500 | 35.5000 |
| 5 | 19.5 | D,E | 4.2760 | 2 | 3.6875 | 0.5885 | 16.4000 | 32.8000 |
| 5 | 25.6 | D,E | 4.0000 | 2 | 3.3750 | 0.6250 | 17.3500 | 34.7000 |
| 5.5 | 21.9 | D,E | 4.7780 | 2 | 4.0000 | 0.7780 | 21.2500 | 42.5000 |
| 5.5 | 24.7 | D,E | 4.6700 | 2 | 4.0000 | 0.6700 | 18.5200 | 37.0400 |
| 3.5 | 15.5 | X,G,S | 2.6020 | 2 | 1.9375 | 0.6645 | 18.3800 | 36.7600 |
| 4.5 | 20 | X,G,S | 3.6400 | 2 | 2.8125 | 0.8275 | 22.4800 | 44.9600 |
| **5** | **19.5** | **X,G,S** | **4.2760** | **2** | **3.5625** | **0.7135** | **19.6300** | **39.2600** |
| 5 | 25.6 | X,G,S | 4.0000 | 2 | 3.3125 | 0.6875 | 18.9600 | 37.9200 |
| 5.5 | 21.9 | X,G,S | 4.7780 | 2 | 3.8125 | 0.9655 | 25.7700 | 51.5400 |
| 5.5 | 24.7 | X,G,S | 4.6700 | 2 | 3.8125 | 0.8575 | 23.2100 | 46.4200 |

***θ≤45°***

$$K_e = 2.6\sin\left(\frac{\theta}{2}\right)(1-\beta^2)^2 = 2.6\sin\left(\frac{39.26}{2}\right)\left[1-\left(\frac{3}{4.276}\right)^2\right]^2 = 0.2276.$$

Where β is the ratio of diameters of small to large pipes, dimensionless

$\beta = d_{TJ}/D_p$

Note: internal diameter of pipe for this example is 4.276 because is the data that we have been cover in table C1.

The mechanical energy loss, *Fe*, between two different successive diameters can be expressed by comparing the Bernoulli equation at two points. See Eq.7.2.

$$v = \frac{0.408q}{d_{TJ}^{\ 2}} = \frac{0.408 \times 100}{(3)^2} = 4.533 \text{ ft/sec}$$

$$F_e = K_e\left(\frac{v^2}{2gc}\right) = 0.2276\left(\frac{4.533^2}{2 \times 32.2}\right) = 0.07263 \text{ lbf ft/lbm.}$$

The pressure loss then is calculated by multiplying the fluid density with mechanical energy loss for gradual enlargements.

$\Delta p_e = 0.052 F_e \rho = 0.052 \text{x} 0.07263 \text{x} 11.55 = 0.0436$ psi.

**Gradual Contraction**

***θ≤45°***

$$K_c = 0.8\sin\left(\frac{\theta}{2}\right)(1-\beta^2) = 0.8\sin\left(\frac{39.26}{2}\right)\left(1-\left(\frac{3}{4.276}\right)^2\right) = 0.1365$$

Where β is the ratio of diameters of small to large pipes, dimensionless

$\beta = d_{TJ}/D_p$

Then;

$$F_c = K_c\left(\frac{v^2}{2gc}\right) = 0.1365\left(\frac{4.533^2}{2 \times 32.2}\right) = 0.04355 \text{ lbf ft/lbm}$$

$\Delta p_c = 0.052 F_c \rho = 0.052 \text{x} 0.04355 \text{x} 11.55 = 0.02616$ psi

Note the convergence or divergence angle can be estimated using tables and figures in Chapter VI. Also, see Fig. 7.3 and Tables 7.1, 7.2.

- Annulus

**Gradual Enlargement and Contraction**

The procedure is the same followed in Sections 7.1.1 and 7.1.2. However, notice that the velocity used to estimate the pressure loss by enlargement and contraction corresponds to the narrow annulus. See Fig. 7.4.

**Table C2— Angles for External Upset (Annulus) for Given Example**

| Pipe | | | | tool joint | | | angle calculation | |
|---|---|---|---|---|---|---|---|---|
| **D,in** | **wt(lbf/ft)** | **Grade** | **d,in** | **meu,in** | **Dou,in** | **Dou-D** | **θ/2** | **θ** |
| 4.5 | 20 | D,E | 3.6400 | 1.5 | 4.7810 | 0.2810 | 10.6100 | 21.2200 |
| 5 | 19.5 | D,E | 4.2760 | 1.5 | 5.1880 | 0.1880 | 7.1400 | 14.2800 |
| 5 | 25.6 | D,E | 4.0000 | 1.5 | 5.1880 | 0.1880 | 7.1400 | 14.2800 |
| 5.5 | 21.9 | D,E | 4.7780 | 1.5 | 5.5630 | 0.0630 | 2.4000 | 4.8000 |
| 5.5 | 24.7 | D,E | 4.6700 | 1.5 | 5.5630 | 0.0630 | 2.4000 | 4.8000 |
| 3.5 | 15.5 | X,G,S | 2.6020 | 2.5 | 3.7810 | 0.2810 | 6.4100 | 12.8200 |
| 4.5 | 20 | X,G,S | 3.6400 | 2.5 | 4.7810 | 0.2810 | 6.4100 | 12.8200 |
| **5** | **19.5** | **X,G,S** | **4.2760** | **2.5** | **5.1880** | **0.1880** | **4.3000** | **8.6000** |
| 5 | 25.6 | X,G,S | 4.0000 | 2.5 | 5.1880 | 0.1880 | 4.3000 | 8.6000 |
| 5.5 | 21.9 | X,G,S | 4.7780 | 2.5 | 5.5630 | 0.0630 | 1.4400 | 2.8800 |
| 5.5 | 24.7 | X,G,S | 4.6700 | 2.5 | 5.5630 | 0.0630 | 1.4400 | 2.8800 |

***θ≤45°***

$$K_e = 2.6\sin\left(\frac{\theta}{2}\right)(1-\beta^2)^2 = 2.6\sin\left(\frac{8.6}{2}\right)\left[1-\left(\frac{6.75}{10.711}\right)^2\right]^2 = 0.07086$$

Where β is the ratio of diameters of small to large pipes, dimensionless

$\beta = D_{TJ}/D_2$

The mechanical energy loss, *Fe*, between two different successive diameters can be expressed by comparing the Bernoulli equation at two points. See Eq.7.2

$$v_a = \frac{0.408q}{(D_2{}^2 - D_{TJ}^2)} = \frac{0.408\times 100}{(10.711^2 - 6.75^2)} = 0.5899 \text{ ft/sec}$$

$$F_e = K_e\left(\frac{v^2}{2gc}\right) = 0.07086\left(\frac{0.5899^2}{2\times 32.2}\right) = 0.00038 \text{ lbf ft/lbm.}$$

The pressure loss is then calculated by multiplying the fluid density with mechanical energy loss for gradual enlargements.

$\Delta p_e = 0.052 F_e \rho.$ =0.052x0.00038x11.55 = 0.00023 psi.

**Gradual Contraction**

***θ≤45°***

$$K_c = 0.8\sin\left(\frac{\theta}{2}\right)(1-\beta^2) = 0.8\sin\left(\frac{8.6}{2}\right)(1-0.63^2) = 0.036$$

Where β is the ratio of diameters of small to large pipes, dimensionless

$\beta = D_{TJ}/D_2$

Then;

$$F_c = K_c\left(\frac{v^2}{2gc}\right) = 0.036\left(\frac{0.5899^2}{2\times 32.2}\right) = 0.0002 \text{ lbf ft/lbm}$$

$\Delta p_c = 0.052 F_c \rho = 0.052$x0.0002x11.55=0.00012 psi

## C-1.1 Estimation of Pump Pressure Considering Enlargement and Contraction correction

Add to drillstring friction pressure losses calculated (with any correction) the pressure losses caused by enlargement and contraction of each tool joint. Do the same for the annulus friction pressure losses. See Eq. 7.9. Table B1 shows the values for pressure drop in the annulus and in the drillstring for Newtonian fluid.

$$\Delta p_p = \Delta p_s + [\Delta p_{ds} + (\Delta p_{e,} + \Delta p_c) N_{TJ}] + [\Delta p_a + (\Delta p_e + \Delta p_c) N_{TJ}] + \Delta p_b.$$

Where $N_{TJ}$ =12440ft/30ft= 415

$\Delta p_p = 0 + [47.84 + (0.0436 + 0.02616) \times 415] + [8.24 + (0.00023 + 0.00012) \times 415] + 3.257 = 88.6$ psi

**C-2 Equivalent Diameter**

Use the following equation to estimate equivalent diameter, $D_e$, in drillstring (between inside pipe and tool joint diameters). [31]

- Pipe

$$De_p = \left[ \frac{L_2 {d_{TJ}}^4 D_P^4}{L_1 D_P^4 + (L_1 - L_2) d_{TJ}^4} \right]^{1/4} = \left[ \frac{21 \times 3^4 \times 4.276^4}{21 \times 4.276^4 + (21 - 336) \times 3^4} \right]^{1/4} = 4.089 \text{ in}$$

Where:

$d_{TJ}$ =3 in.

$D_P$= 4.276 in.

$L_1$ = 21 in, from drilling manual (i.e. pin + box tong length).

$L_2$= 28 ft x 12= 336 in. (length of one pipe without tool joint)

- Annulus

$$De_a = \left[ \frac{L_2 {D_{TJ}}^4 D_1^4}{L_1 D_1^4 + (L_1 - L_2) D_{TJ}^4} \right]^{1/4} = \left[ \frac{21 \times 6.75^4 \times 5^4}{21 \times 5^4 + (21 - 336) \times 6.75^4} \right]^{1/4} = 5.06 \text{ in}$$

Where:

$D_{TJ}$ = 6.75 in.

$D_1$= 5 in.

Finally, calculate friction pressure losses in the drillstring and annulus as showed in appendix B, but use equivalent diameter in the calculation of frictional pressure drop.

Note: use $D_p = De_p$ for the estimation of pressure drop in the drillstring and $D_2 = De_a$ to estimate pressure drop in the annulus.

**$q_1$=100 gallon/min**

- Pipe Flow

$$v_p = \frac{0.408q}{D_p^{\,2}} = \frac{0.408\times 100}{(4.089)^2} = 2.4402 \text{ ft/sec}$$

$\mu_a$= $R_{300}$= 58 cp

$$N_{\mathrm{Re}} = \frac{928 De_p v_p \rho}{\mu_a} = \frac{928\times 4.089\times 2.4402\times 11.55}{58} = 1843.93$$

If $N_{\mathrm{Re}} < N_{\mathrm{Rec}} \rightarrow$flow is laminar.

$f$= 16/ $N_{\mathrm{Re}}$ = 16/ 1843.93=0.008677

$$\left(\frac{dp}{dL}\right) = \frac{f v_p^{\,2} \rho}{25.81 De_p} = \frac{0.008677\times (2.4402)^2 \times 11.55}{25.81\times 4.089} = 0.0057 \text{ psi/ft}$$

$$\Delta p_{ds} = \left(\frac{dp}{dL}\right)\Delta L = 0.0057\times 12440 = 70.343 \text{ psi}$$

- Annular Flow

$$v_a = \frac{0.408q}{(D_2^{\,2} - De_a^2)} = \frac{0.408\times 100}{(10.711^2 - 5.06^2)} = 0.4578 \text{ ft/sec}$$

$$N_{\mathrm{Re}} = \frac{757(D_2 - De_a) v_a \rho}{\mu_a} = \frac{757\times (10.711 - 5.06)\times 0.4578\times 11.55}{58} = 389.988$$

If $N_{\mathrm{Re}} < N_{\mathrm{Rec}} \rightarrow$ flow is laminar.

$f = 16/N_{Re} = 16/389.988 = 0.04103$

$$\left(\frac{dp}{dL}\right) = \frac{f v_a^{\,2} \rho}{25.81(D_2 - De_a)} = \frac{0.04103 \times (0.4578)^2 \times 11.55}{25.81 \times (10.711 - 5.06)} = 0.00068 \quad \text{psi/f}t$$

$$\Delta p_a = \left(\frac{dp}{dL}\right)\Delta L = 0.00068 \times 12440 = 8.471 \text{psi}$$

- Frictional pressure losses across the bit, $\Delta p_b$:

$$\Delta p_b = \frac{156 \rho q^2}{(D_{N1}^{\,2} + D_{N2}^{\,2} + D_{N3}^{\,2})^2} = \frac{156 \times 11.5 \times 100^2}{(28^2 + 28^2 + 28^2)^2} = 3.257 \text{ psi}$$

$\Delta p_p = 0 + 70.343 + 3.257 + 8.471 = 82.07$ psi

**C-3 Two Different IDs**

This proposed approach estimate the frictional pressure drop in the annulus and in the drillstring considering the actual pipe/tool joint length and diameter in the calculation. Follow the example in appendix B.

- Pipe:

a. Estimation of total drillstring length, $L_{\text{total } dp}$:

$L_{\text{total } dp} = (L_2 N_{DP} - L_1 N_{TJ}) = (12440 - 21 \text{x} 415/12) = 11713.75$ ft

b. Use the inner diameter of the drillstring to estimate the frictional pressure drop, $\Delta p_{ds}$:

$$\Delta p_{ds} = \left(\frac{dp}{dL}\right)\Delta L = 0.0038 \text{ x } 11713.75 = 44.51 \text{ psi}$$

Where:

$L_{\text{total } dp}$= total drill pipe length, ft

$N_{DP}$=numbers of drillpipe

$\Delta p_{ds}$ = frictional pressure drop in the drillstring, psi.

$(dp/dL)_{ds}$ =pressure gradient, psi/ft. See Chapter V.

c. Estimation of total tool joint length, $L_{\text{total } TJ}$:

$L_{\text{total } TJ}$ = 21x415/12= 726.25 ft

d. Use ID of tool joint and respective length to calculate its contribution in the pressure loss to the drillstring, $\Delta p_{TJ}$.

$$v_p = \frac{0.408q}{D_p^{\ 2}} = \frac{0.408 \times 100}{(3)^2} = 4.533 \text{ ft/sec}$$

$\mu_a$= $R_{300}$= 58 cp

$$N_{\text{Re}} = \frac{928 d_{TJ} v_{TJ} \rho}{\mu_a} = \frac{928 \times 3 \times 4.533 \times 11.55}{58} = 2513.09$$

If $N_{\text{Re}} < N_{\text{Re}c} \rightarrow$flow is turbulent.

$f$= 0.0791/ $N_{\text{Re}}^{0.25}$= 0.0791/ $(2513.09)^{0.25}$=0.01117

$$\left(\frac{dp}{dL}\right) = \frac{f v_p^{\ 2} \rho}{25.81 D_p} = \frac{0.0117 \times (4.533)^2 \times 11.55}{25.81 \times 3} = 0.00343 \text{ psi/ft}$$

$$\Delta p_{TJ} = \left(\frac{dp}{dL}\right)\Delta L = 0.0343 \text{ x } 726.25 = 24.91 \text{ psi}$$

e. Add drillstring and tool joint frictional pressure drop to estimate the total drillstring friction pressure losses.

$(\Delta p_{\text{Total 2IDs}})_{ds}$ = 44.51 +24.91=69.42 psi

- Annulus:

Use the same procedure to estimate the total frictional pressure drop in the annulus, $(\Delta p_{\text{Total 2IDs}})_a$, followed in the pipe section but use the annulus data.
Finally,

$\Delta p_p$=0+ 69.42 + 10.25 +3.257=82.93 psi

**C-4 Enlargement and Contraction Plus Equivalent Diameter**

$\Delta p_p = 0 + [70.25 + (0.044 + 0.02624) \times 415] + [10.27 + (0.00023 + 0.00012) \times 415] + 3.257 = 113.072$ psi

**C-5 Enlargement and Contraction plus Two Different IDs**

$\Delta p_p = 0 + [69.42 + (0.044 + 0.02624) \times 415] + [10.25 + (0.00023 + 0.00012) \times 415] + 3.257 = 113.072$ psi

$\Delta p_p = 0 + 69.42 + 10.25 + 3.257 = 112.22$ psi

# APPENDIX D

## Table D1— Rheological Properties for Eight Rheological Models

Mud density, ppg= 11.55 P,psi= 14.7
Mud type: SBM-oil T, F= 150

| $\gamma_N$ ($sec^{-1}$) | $\tau_{fann}$ | $\tau_{new}$ | $\tau_{B-M\ (API13RP)}$ | $\tau_{P-L}$ | $\tau_{API}$ | $\tau_{H-B(full\ range)}$ | $\tau_{Casson}$ | $\tau_{R-S}$ | $\tau_{Unified}$ |
|---|---|---|---|---|---|---|---|---|---|
| 1021.80 | 98.16 | 108.92 | 98.16 | 85.46 | 98.17 | 100.09 | 101.16 | 96.44 | 97.48 |
| 510.90 | 61.89 | 54.46 | 61.89 | 62.65 | 61.89 | 63.66 | 61.78 | 63.45 | 61.47 |
| 340.60 | 49.08 | 36.31 | 49.79 | 52.25 | 47.25 | 49.35 | 47.42 | 49.87 | 46.94 |
| 170.30 | 34.14 | 18.15 | 37.70 | 38.30 | 34.17 | 32.70 | 31.56 | 33.43 | 30.93 |
| 10.22 | 10.67 | 1.09 | 26.33 | 10.86 | 11.23 | 10.16 | 10.92 | 10.16 | 9.55 |
| 5.11 | 8.54 | 0.54 | 25.97 | 7.97 | 8.54 | 8.80 | 9.52 | 8.94 | 8.30 |
| AAPE | | 46.538 | 60.48599 | 6.8871 | 1.512089 | 2.8978288 | 4.66646946 | 2.91378324 | 4.73638305 |

| Newt. | | B-M(API13RP) | | P-L(full range) | | API(P-L) | | H-B(full range) | | Casson | | R-S | | Unified | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $\mu a_{6\ points}$= | 51.04 | $\mu p_{API}$= | 0.0710 | n= | 0.45 | np | 0.67 | n= | 0.7129 | $\mu pc$= | 0.0551 | B= | 0.6186 | np= | 0.67 |
| $\mu a_{1\ point}$= | 58.00 | $\tau y_{API}$= | 25.61 | k= | 3.84 | na= | 0.40 | k= | 0.6686 | $\tau oc$= | 6.5238 | A= | 1.3130 | na= | 0.73 |
| | | | | | | kp= | 0.97 | $\tau o$= | 6.66 | | | C= | 17.12 | kp= | 0.97 |
| | | | | | | ka= | 4.45 | | | | | | | ka= | 0.58 |
| | | | | | | | | | | | | | | LSYP= | 6.40 |
| | | | | | | | | | | | | | | R= | 0.27 |

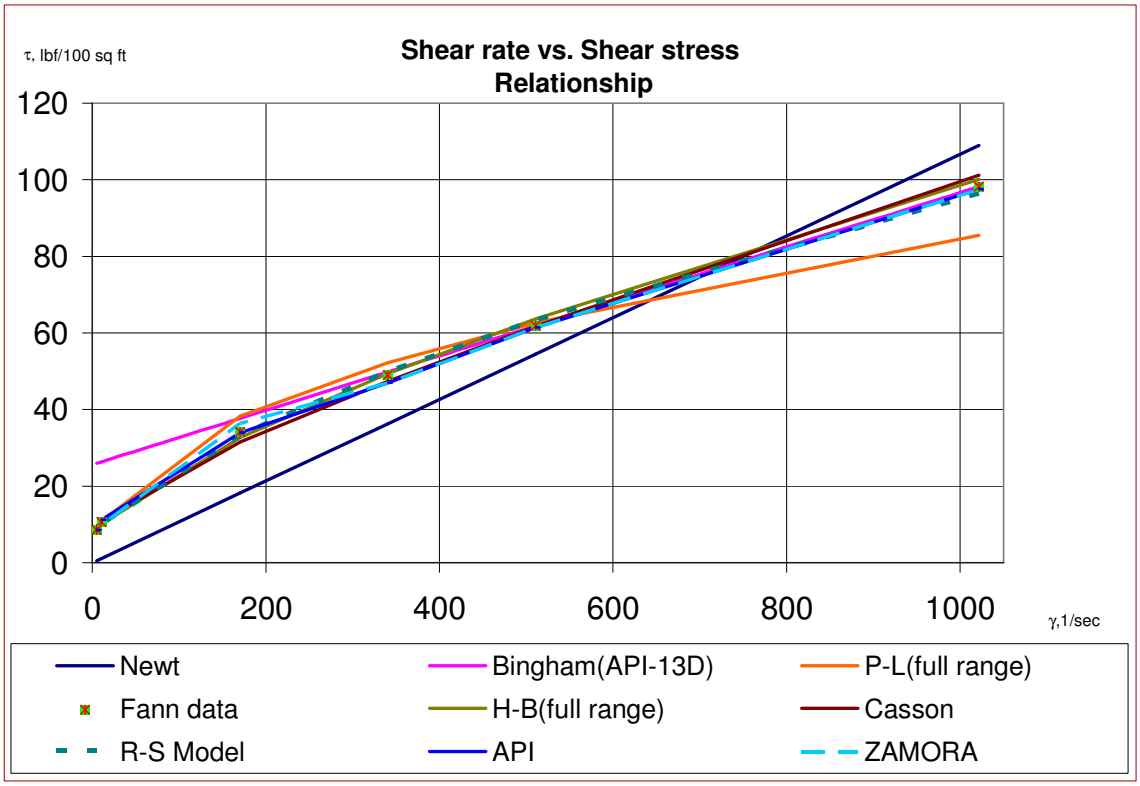



Fig. D1— Rheograms for eight rheological models at 150°F.

# APPENDIX E

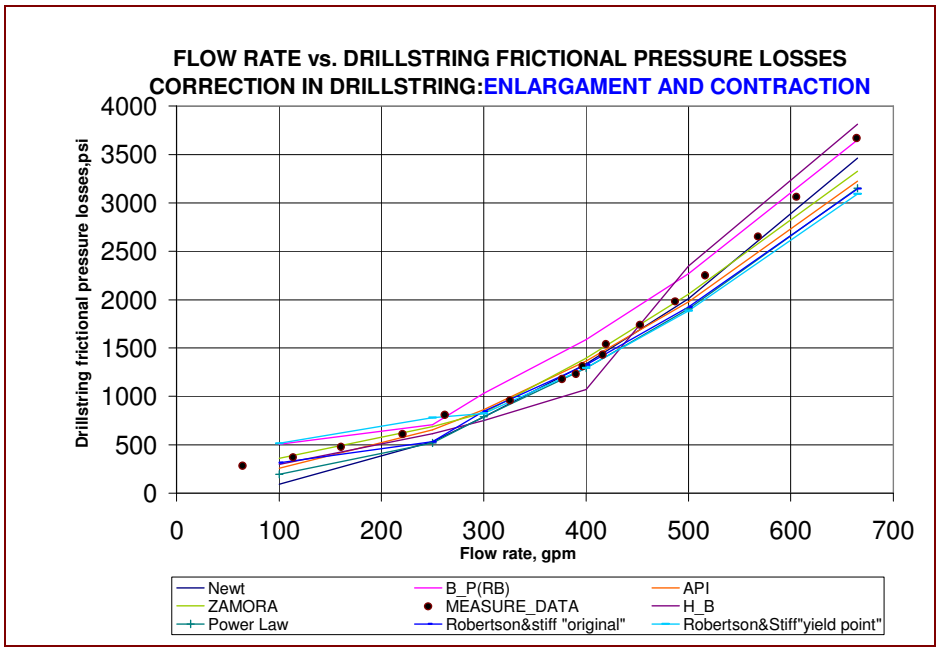



Fig. E1— Flow rate vs. drillstring pressure loss with E&C correction.

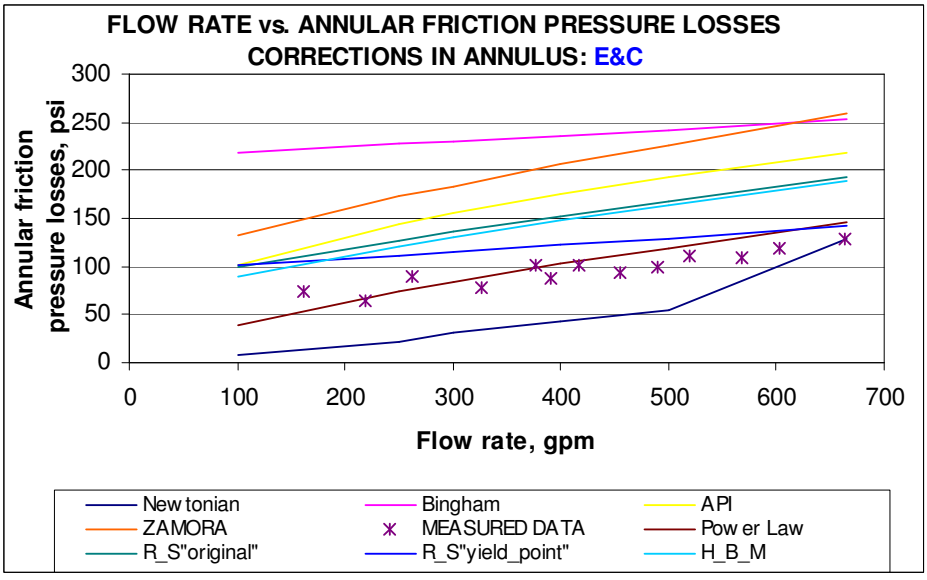



Fig. E2— Flow rate vs. annular pressure loss with E&C correction.

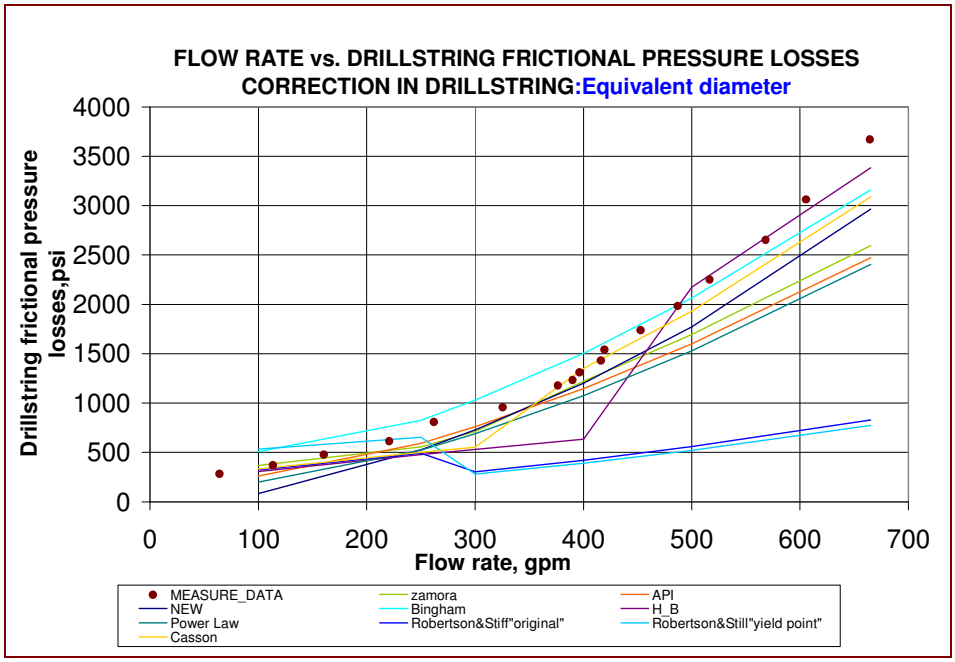



Fig. E3— Flow rate vs. drillstring pressure loss with ED correction.

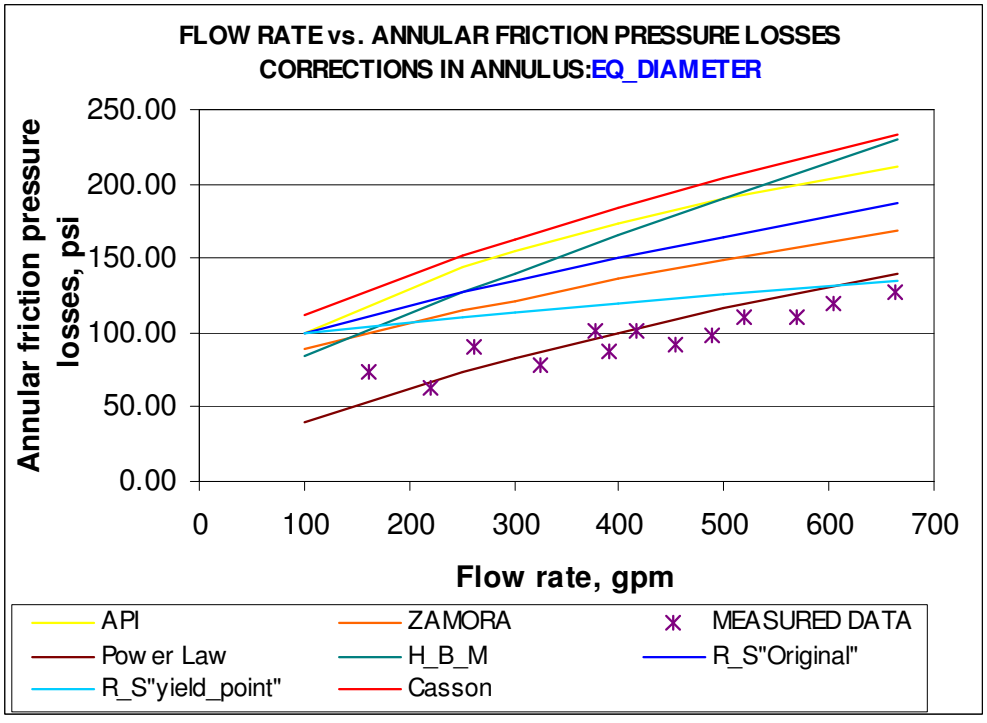



Fig. E4— Flow rate vs. annulus pressure loss with ED correction.

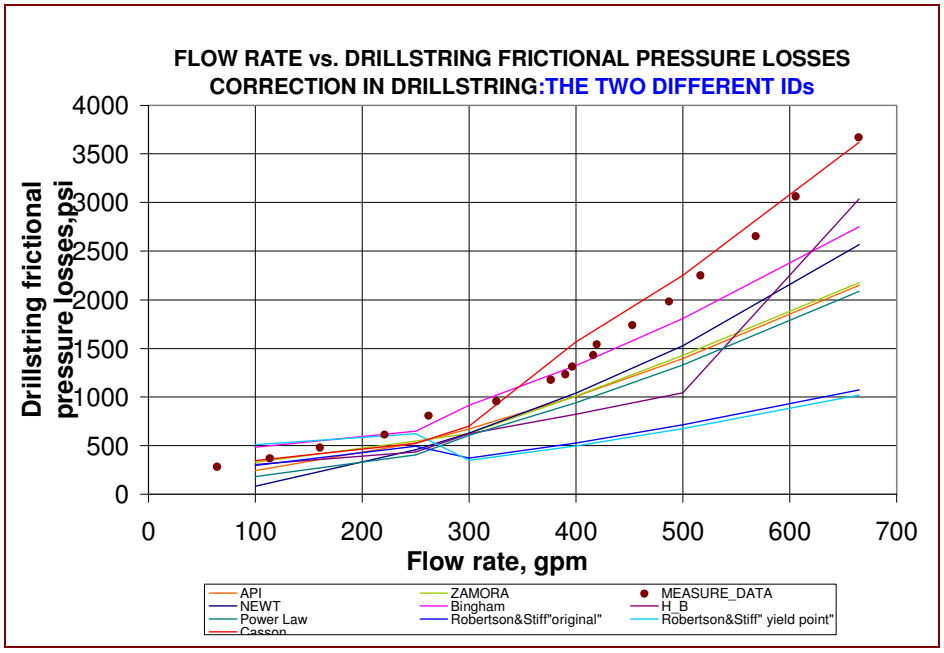



Fig. E5— Flow rate vs. drillstring pressure loss with 2IDs correction.

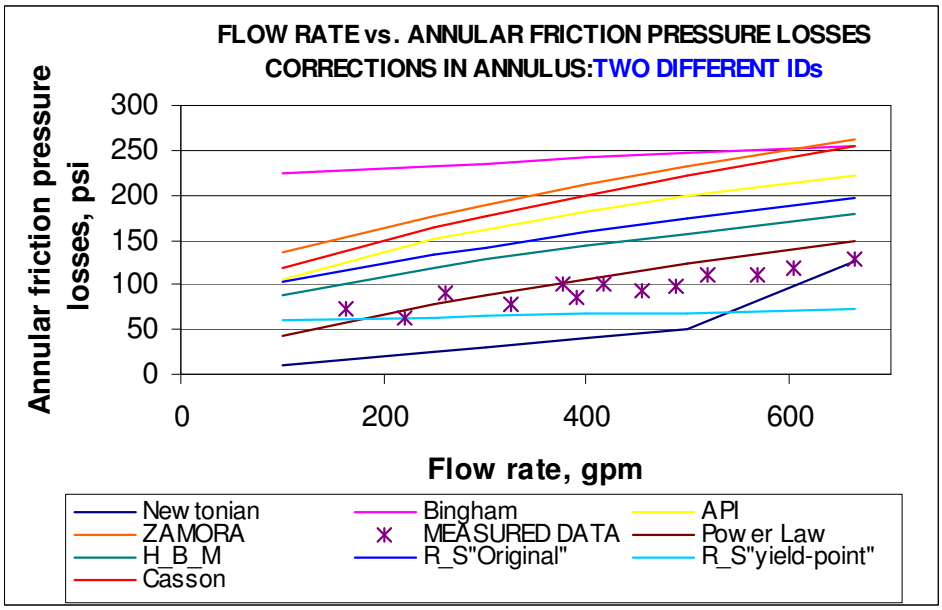



Fig. E6— Flow rate vs. annular pressure loss with correction for 2IDs.

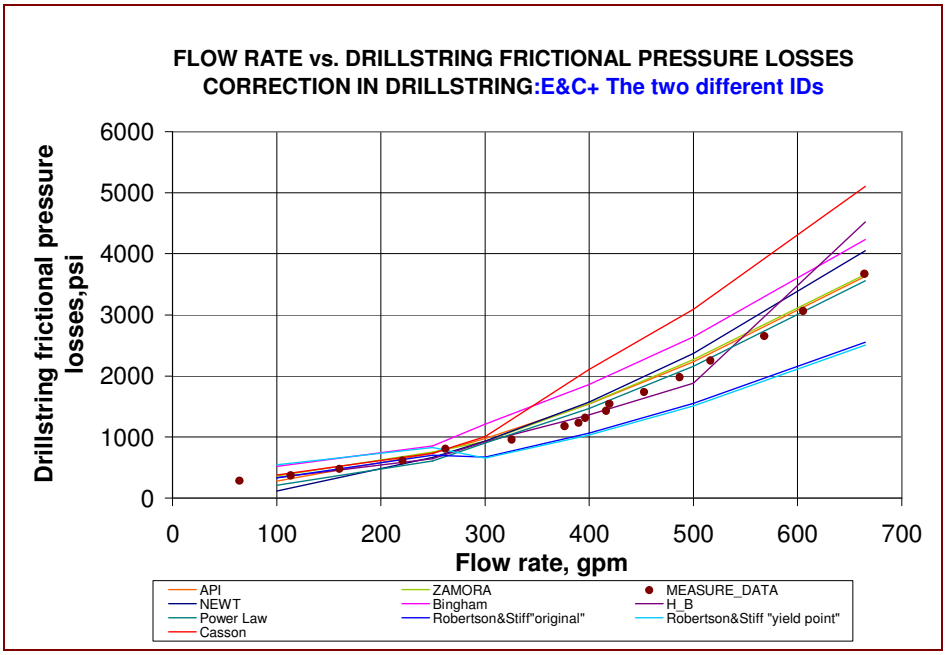



Fig. E7— Flow rate vs. drillstring pressure loss with correction for E&C+2IDs.

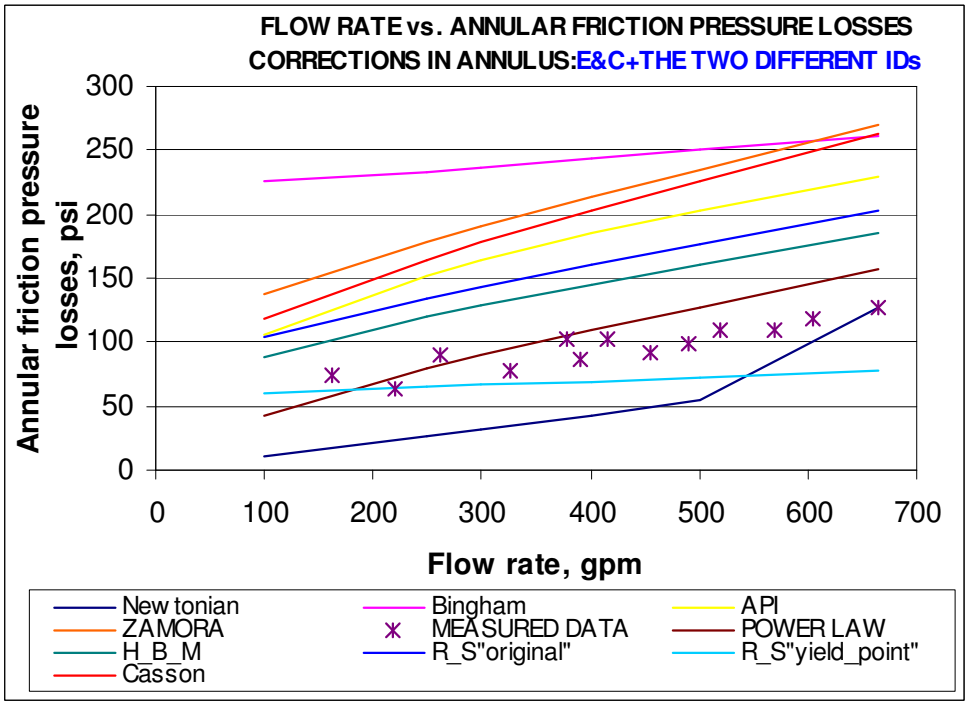



Fig. E8— Flow rate vs. annular pressure loss with correction for E&C+2IDs.

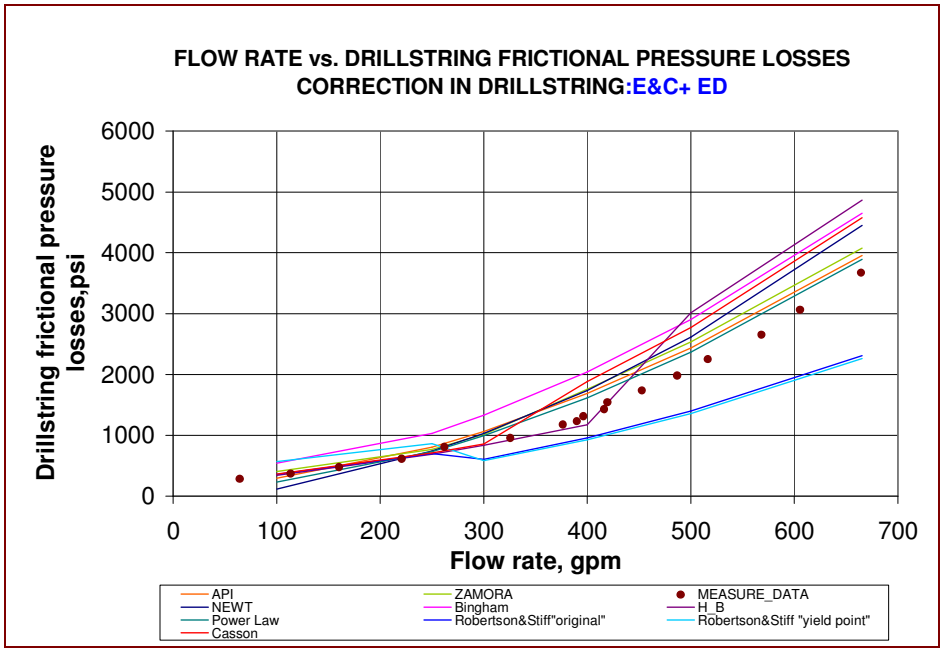



Fig. E9—Flow rate vs. drillstring pressure loss with correction for E&C+ ED.

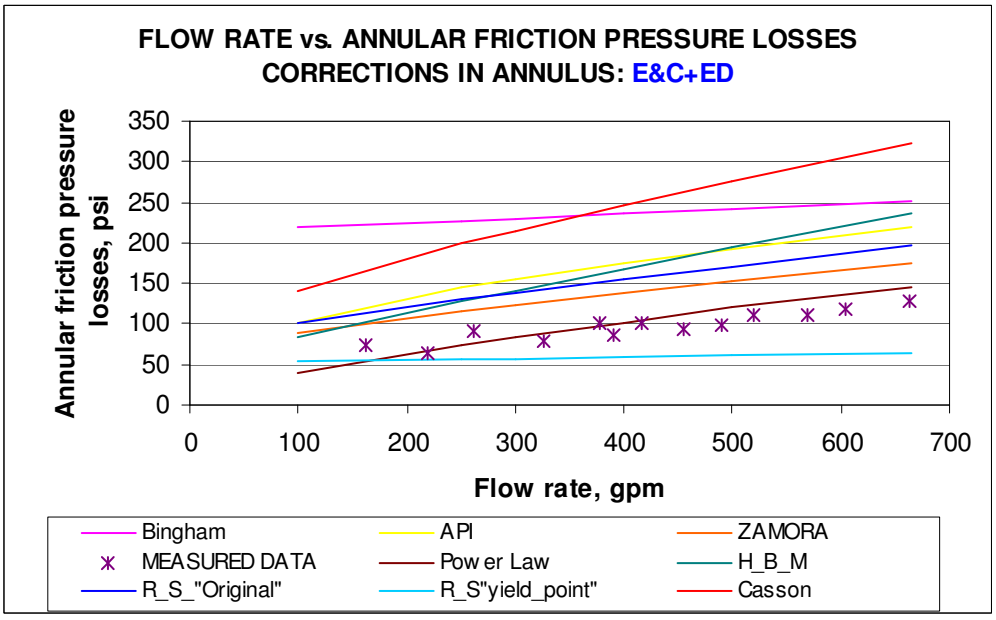



Fig. E10— Flow rate vs. annular pressure loss with correction for E&C+ ED.

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